# The energy cycle of a gasoline engine

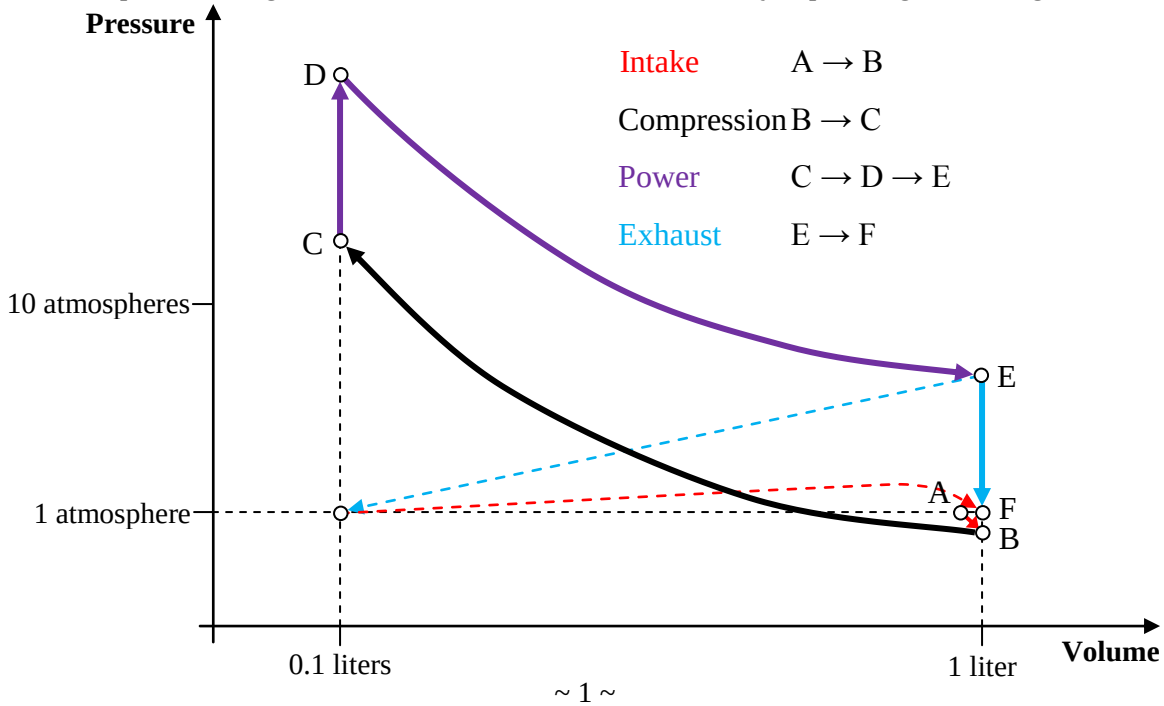
A rule of thumb is used in the selection of turbochargers for gasoline engines: gasoline engines produce 9.5 to 10.5 horsepower at the flywheel for every pound per minute of air flow. The purpose of this paper is to track through the energy cycle of a gasoline engine to see why this is so. Our calculations result in a figure of 9.65 HP per pound/minute. It is amazing that the rule of thumb is so broadly-applicable. It does not depend on the size of the cylinders, or the number of cylinders, or even the engine's RPM.

The engine we will look at is a traditional naturally-aspirated gasoline engine. By "traditional", we mean that a combined mixture of air and gasoline vapour is drawn into the cylinder. The alternative, seen in some very modern engines, is the direct injection of the gasoline, not unlike a diesel engine. By "naturally-aspirated", we mean that the mixture is drawn into the cylinder naturally as the piston head descends and tends to create a vacuum. The alternative, also becoming more common in cars, is to use a turbocharger to pressurize the air in the intake chamber, forcing it into the cylinder whenever the intake valve is open.

In our numerical example, we will use a cylinder whose volume is one liter at bottom-dead-center. We will assume a 10:1 compression ratio, so the cylinder volume will be reduced to 0.1 liters when the piston is at top-dead-center. Compression ratios for gasoline engines are almost always in the range of 9 to 10.5, for reasons we will discuss near the end of this paper.

In an earlier paper titled *The ideal gas law and a bit of thermodynamics*, we described how a graph of pressure versus volume could be used to keep track of the state of a gas as it is transformed by some thermal process. The "state" of an ideal gas is a statement of its physical properties at a particular time. When the quantity of gas is constant during a process, as is the case for the gasoline-air mixture trapped inside a cylinder, there are three physical properties of interest. They are the pressure, volume and temperature. We will assume that the mixture is "homogeneous" at every instant during the process, which means that pressure and temperature are the same at every spot throughout the volume. This is not necessarily a good assumption – some poorly-designed engines suffer from hot spots and cold spots.

The following sketch shows the pressure and volume of the fixed quantity of gasoline-air mixture in a cylinder as it passes through the four strokes of a traditional naturally-aspirated gasoline engine.



**The intake stroke**

The gasoline-air mixture is sucked into the cylinder as the piston head descends. We'll start our analysis with the intake stroke because it determines what gases are inside the cylinder during the rest of the power cycle. To make things interesting, we are going to include some humidity in the ambient air and also to account for the loss of pressure due to suction during the intake stroke.

Let's begin with air that has the pressure and temperature of the U.S. Standard Atmosphere at sea level. Tables for the U.S. Standard Atmosphere give a pressure of 101,325 N/m$^2$ at a temperature of 15°C. This is for clean dry air at a typical Canadian temperature. But let's also assume that the relative humidity on this day is 20%. The relative humidity is the amount of water vapour in the air, stated as a percentage of the maximum amount of water vapour which air at this pressure and temperature can hold. (If there is more water vapour than this maximum, it will condense and fall as rain.) Tables on the website www.EngineeringToolBox.com give a value of 16.7 millibar, or 1,670 N/m$^2$, for the saturation water vapour pressure at this temperature. The actual water vapour pressure will be 20% of this, or 334 N/m$^2$, so the partial pressures which make up the real air on this day are:

| Component gas | Partial pressure |
|---|---|
| Clean dry air | 100,991 N/m$^2$ |
| Water vapour | 334 N/m$^2$ |
| Total for real air | 101,325 N/m$^2$ |

The clean dry air and the water vapour can be treated as ideal gases. When they are mixed, the mixed gas can also be treated as an ideal gas. The Ideal Gas Law ($P \times V = n \times R \times T$) can therefore be used for each component gas and for the mixed gas as well. Once the component gases are mixed, the volume $V$, temperature $T$ (15°C = 288.15°K) and Ideal Gas Constant $R$ (8.314 J/mole-°K) will be the same for both, and the number of moles of the component gases will be proportional to their partial pressures. For example, one cubic meter of real air on this day will contain the following number of moles of molecules:

| Component gas | Number of moles in one cubic meter = $(\boldsymbol{P} \times \boldsymbol{V})/(\boldsymbol{R} \times \boldsymbol{T})$ |
|---|---|
| Clean dry air | $(100{,}991 \times 1)/(8.314 \times 288.15) = 42.155$ moles |
| Water vapour | $(334 \times 1)/(8.314 \times 288.15) = 0.139$ moles |
| Total for real air | $(101{,}325 \times 1)/(8.314 \times 288.15) = 42.295$ moles |

As always, the number of moles can be multiplied by Avogadro's number ($6.0221413 \times 10^{23}$) to convert the number of moles into the number of individual molecules. This is done in the following table.

| Component gas | Number of molecules in one cubic meter of real air |
|---|---|
| Clean dry air | $42.155 \times 6.0221413 \times 10^{23} = 2{,}538.66 \times 10^{22}$ molecules |
| Water vapour | $0.140 \times 6.0221413 \times 10^{23} = 8.40 \times 10^{22}$ molecules |
| Total for real air | $42.295 \times 6.0221413 \times 10^{23} = 2{,}547.06 \times 10^{22}$ molecules |

We need to get more precise about the clean dry air and describe its component gases separately. We can take over results from the earlier paper in which the component gases which make up clean dry air are listed by their relative volumes. The following table shows what makes up one cubic meter of real air, including the humidity. (These figures are also taken from the U.S. Standard Atmosphere.)

| Component gas | % by volume of dry air | Number of molecules in one cubic meter of real air |
|---|---|---|
| Nitrogen ($N_2$) | 78.084% | $1{,}982.30 \times 10^{22}$ molecules |
| Oxygen ($O_2$) | 20.9476% | $531.79 \times 10^{22}$ molecules |
| Argon (Ar) | 0.934% | $23.71 \times 10^{22}$ molecules |
| Carbon dioxide ($CO_2$) | 0.0314% | $0.80 \times 10^{22}$ molecules |
| Neon (Ne) | 0.001818% | $0.05 \times 10^{22}$ molecules |
| Helium (He) | 0.000524% | $0.01 \times 10^{22}$ molecules |
| Methane ($CH_4$) | 0.0002% | $\approx 0$ |
| Total for dry air | 100% | $2{,}538.66 \times 10^{22}$ molecules |
| Water vapour | | $8.40 \times 10^{22}$ molecules |
| Total for real air | | $2{,}547.06 \times 10^{22}$ molecules |

All of these gases other than the oxygen are carried through the entire power cycle without making any contribution. From the point-of-view of efficiency, they are simply dead weight which must be endured to allow us to use the oxygen in real air.

We are going to assume that the gasoline we burn is 100% octane. We are also going to assume that it is mixed with real air at the exact stoichiometric ratio. From an earlier paper titled *The stoichiometry of burning hydrocarbon fuels*, we know that perfect combustion occurs when the eight atoms of carbon in each molecule of octane marry with the 25 atoms of oxygen in 12½ molecules of oxygen to produce nine molecules of water vapour and eight molecules of carbon dioxide. This is the reaction:

$$C_8H_{18} + 12½O_2 \rightarrow 9H_2O + 8CO_2$$

Expressed in terms of the $531.79 \times 10^{22}$ molecules of oxygen in the above table, the reaction is:

$$42.54 \times 10^{22} C_8H_{18} + 531.79 \times 10^{22} O_2 \rightarrow 382.89 \times 10^{22} H_2O + 340.35 \times 10^{22} CO_2$$

We can re-write the above table in two ways, once to include the octane vapour before combustion and a second time to show the byproducts after combustion.

| Component gas | Number of molecules (NOT IN ONE CUBIC METER) | | | |
|---|---|---|---|---|
| | Before combustion | | After combustion | |
| Nitrogen | $1{,}982.30 \times 10^{22}$ molecules | 76.5484% | $1{,}982.30 \times 10^{22}$ molecules | 72.3862% |
| Oxygen | $531.79 \times 10^{22}$ molecules | 20.5356% | 0 | |
| Octane | $42.54 \times 10^{22}$ molecules | 1.6429% | 0 | |
| Argon | $23.71 \times 10^{22}$ molecules | 0.9156% | $23.71 \times 10^{22}$ molecules | 0.8658% |
| Carbon dioxide | $0.80 \times 10^{22}$ molecules | 0.0308% | $341.14 \times 10^{22}$ molecules | 12.4573% |
| Neon | $0.05 \times 10^{22}$ molecules | 0.0018% | $0.05 \times 10^{22}$ molecules | 0.0017% |
| Helium | $0.01 \times 10^{22}$ molecules | 0.0005% | $0.01 \times 10^{22}$ molecules | 0.0005% |
| Water vapour | $8.40 \times 10^{22}$ molecules | 0.3242% | $391.29 \times 10^{22}$ molecules | 14.2883% |
| Total | $2{,}589.60 \times 10^{22}$ molecules | 100% | $2{,}738.50 \times 10^{22}$ molecules | 100% |

There are a few things to note:

1. These molecules no longer fit into one cubic meter. The octane in the fuel-air mixture is in gaseous form. Like the other gases in the mixture, it can be treated like an ideal gas. The

addition of octane molecules to the one cubic meter of real air we were dealing with before will tend to displace some of the molecules already there. Unless the volume is increased, the extra molecules will increase the pressure. We will adjust for this in just a moment.

2. The combustion process adds heat to the mixture, as we know. But it also decreases the number of molecules.

3. The byproducts of combustion are water vapour and carbon dioxide. Note that a certain quantity of these two gases is present in the real air even before combustion takes place.

4. Methane is present in such a small quantity that we will not continue to account for it.

We now know the types of molecules in the fuel-air mixture which are sucked into the cylinder during the intake stroke. We also know the relative percentages of these types – these are the percentages given in the "Before combustion" columns of the previous table. We can easily scale the number of molecules so that the total number is consistent with any given volume, such as one cubic meter, at the ambient pressure we are assuming. We calculated above (using the Ideal Gas Law) that one cubic meter of an ideal gas (like the fuel-air mixture) at a pressure of 101,325 $N/m^2$ and a temperature of 15°C = 288.15°K contains 42.295 moles, or $2{,}547.06 \times 10^{22}$ molecules. Scaling the columns in the previous table to this total gives:

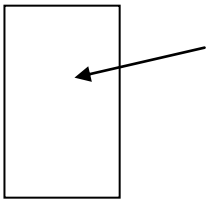
| **Component gas** | **Number of molecules in one cubic meter of fuel-air mixture** | | | |
|---|---|---|---|---|
| | **Before combustion** | | **After combustion** | |
| Nitrogen | $1{,}949.73 \times 10^{22}$ molecules | 76.5484% | $1{,}949.73 \times 10^{22}$ molecules | 72.3862% |
| Oxygen | $523.05 \times 10^{22}$ molecules | 20.5356% | 0 | |
| Octane | $41.84 \times 10^{22}$ molecules | 1.6429% | 0 | |
| Argon | $23.32 \times 10^{22}$ molecules | 0.9156% | $23.32 \times 10^{22}$ molecules | 0.8658% |
| Carbon dioxide | $0.78 \times 10^{22}$ molecules | 0.0308% | $335.54 \times 10^{22}$ molecules | 12.4573% |
| Neon | $0.05 \times 10^{22}$ molecules | 0.0018% | $0.05 \times 10^{22}$ molecules | 0.0017% |
| Helium | $0.01 \times 10^{22}$ molecules | 0.0005% | $0.01 \times 10^{22}$ molecules | 0.0005% |
| Water vapour | $8.26 \times 10^{22}$ molecules | 0.3242% | $384.86 \times 10^{22}$ molecules | 14.2883% |
| Total | $2{,}547.06 \times 10^{22}$ molecules | 100% | $2{,}693.51 \times 10^{22}$ molecules | 100% |

If the intake chamber of our engine had a volume of one cubic meter (it doesn't), it would contain these molecules. The cylinder we are looking at has a much smaller volume – only one liter, which is one-thousandth of one cubic meter. (A liter is a cube whose sides are ten centimeters long, a little smaller than a quart.) But, we cannot simply take one-thousandth of the numbers of the molecules in this table. There are other factors to consider. The nominal volume of the cylinder is one liter, but the piston head does not sweep out a volume of one liter. Since we are assuming that our engine has a compression ratio of 10:1, the swept volume is only 9/10 of one liter. Furthermore, the timing of the intake valve will never be perfect. There will always be some lead or lag in its openings and closings which prevent 0.9 liter of fuel-air mixture in the intake chamber from being transferred holus-bolus into the cylinder. That the intake process depends on suction further reduces the quantity transferred. The pressure inside the cylinder has to be reduced below the pressure in the intake chamber in order to create the necessary wind of fuel-air mixture into the cylinder.

Furthermore, there is something else in the cylinder as well. There will be some exhaust gas left over from the previous power cycle. During the exhaust stroke which preceded the current intake stroke, the piston head only cleared out 90% of the cylinder. About one-tenth of the exhaust gas from the previous power cycle remains in the cylinder at the start of the current intake stroke. How can we sort all of these factors out?

We are going to assume that a 5% underpressure is required to suck fuel-air mixture from the intake manifold into the cylinder. Similarly, we will assume that a 5% overpressure is required to force exhaust gas out of the cylinder during the exhaust stroke. (Since normal air pressure is approximately 15 psi, this 5% pressure differential is equivalent to 5% × 15 psi = 0.75 psi.)

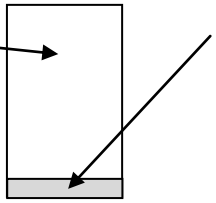
To understand the intake stroke, we have to look back to the end of the previous power stroke, and consider the exhaust gas which was inside the cylinder at that time. Since we (the authors) have already done the analysis all the way through to the end, we can tell you that the state of the exhaust gas at that time was as follows:



Pressure = 592,967 N/m$^2$
Volume = 1 liter
Temperature = 1,698.39°K
Number of moles = 0.041994

We can imagine this exhaust gas as being two distinct parts. The first part is the 90% which will escape through the exhaust valve as the piston head rises during the exhaust stroke. The second part is the 10% which will not leave the cylinder. For modeling purposes, we can represent these two parts as follows:

<u>Gas which will leave cylinder</u>
Pressure = 592,967 N/m$^2$
Volume = 0.9 liter
Temperature = 1,698.39°K
Number of moles = 0.037794



<u>Gas which will remain in cylinder</u>
Pressure = 592,967 N/m$^2$
Volume = 0.1 liter
Temperature = 1,698.39°K
Number of moles = 0.0041994

At this point, in order to understand the intake stroke, we only need to consider the gas which will remain inside the cylinder. At the end of the exhaust stroke, it will consist of the same number of moles (0.0041994 moles) and will occupy the same volume (0.1 liters) as it did at the start of the exhaust stroke. Only its pressure and temperature will have changed. We know the pressure. Including the 5% overpressure, the pressure will be $1.05 \times 101{,}325\ \mathrm{N/m^2} = 106{,}391\ \mathrm{N/m^2}$. Since we know the change in pressure, we can calculate the change in temperature. When we do this (below), we find that the temperature of this part of the exhaust gas will have fallen to 304.73°K = 31.58°C at the end of the exhaust stroke. This may seem too low, but bear in mind that this part of the gas is not what we normally think of as the exhaust gas. That part left the cylinder, at a high temperature. The part which remains in the cylinder cools as the pressure decreases. Incidentally, the 0.004199 moles of exhaust gas left in the cylinder when the intake stroke starts is equivalent to $0.004199 \times 6.0221413 \times 10^{23} = 252.89 \times 10^{19}$ molecules.

Now, let's look ahead to the end of the intake stroke. We're sorry to do so much jumping around. Here is what we are trying to do. We have just calculated the number of molecules of gas left inside the cylinder from the previous power cycle. We are now going to calculate the total number of molecules which must be inside the cylinder at the end of the intake stroke. The difference will be the number of molecules which are sucked in from the intake manifold.

At the end of the intake stroke, the volume of the cylinder will be at its maximum, 0.1 liter, or 0.001 cubic meters. Since there is a 5% underpressure, the gases will be subject to absolute pressure of $0.95 \times 101.325\ \mathrm{N/m^2} = 96{,}259\ \mathrm{N/m^2}$. The temperature of the mixed gas will be a weighted average of the temperatures of the fuel-air mixture coming in from the intake manifold (at 15°C) and the left-over exhaust gas (at 31.58°C). The weighted average temperature we have used is 16.74°C, or 289.89°K.

We now have everything we need to use the Ideal Gas Law to calculate the number of moles of gas inside the cylinder at the end of the intake stroke, as follows:

$$n = \frac{P \times V}{R \times T} = \frac{96{,}259 \times 0.001}{8.314 \times 289.89} = 0.039939 \text{ moles} = 2{,}405.16 \times 10^{19} \text{ molecules}$$

The difference between these quantities ($2{,}405.16 \times 10^{19} - 252.89 \times 10^{19} = 2{,}152.26 \times 10^{19}$) is the number of molecules sucked into the cylinder during the intake stroke. We can take the proportions from the "Before combustion" columns of the previous table to calculate the number of different molecules drawn in during the intake stroke.

| **Component gas** | **Number of molecules drawn into cylinder during the intake stroke** | |
|---|---|---|
| Nitrogen | $1{,}647.52 \times 10^{19}$ molecules | 76.5484% |
| Oxygen | $441.98 \times 10^{19}$ molecules | 20.5356% |
| Octane | $35.36 \times 10^{19}$ molecules | 1.6429% |
| Argon | $19.71 \times 10^{19}$ molecules | 0.9156% |
| Carbon dioxide | $0.66 \times 10^{19}$ molecules | 0.0308% |
| Neon | $0.04 \times 10^{19}$ molecules | 0.0018% |
| Helium | $0.01 \times 10^{19}$ molecules | 0.0005% |
| Water vapour | $6.98 \times 10^{19}$ molecules | 0.3242% |
| Total | $2{,}152.26 \times 10^{19}$ molecules | 100% |

We (the authors) also know the composition of the gas left inside the cylinder at the end of the exhaust stroke. These molecules are the following.

| **Component gas** | **Number of molecules left in the cylinder after the exhaust stroke** | |
|---|---|---|
| Nitrogen | $183.06 \times 10^{19}$ molecules | 72.3862% |
| Oxygen | 0 | |
| Octane | 0 | |
| Argon | $2.19 \times 10^{19}$ molecules | 0.8658% |
| Carbon dioxide | $31.50 \times 10^{19}$ molecules | 12.4573% |
| Neon | $\approx 0$ | 0.0017% |
| Helium | $\approx 0$ | 0.0005% |
| Water vapour | $36.13 \times 10^{19}$ molecules | 14.2883% |
| Total | $252.89 \times 10^{19}$ molecules | 100% |

We can combine the molecules from the two sources (set out separately in the two previous tables) to list the number of molecules inside the cylinder at the end of the intake stroke, as follows:

| **Component gas** | **Number of molecules drawn into cylinder during the intake stroke** | **Number of molecules left in the cylinder after the exhaust stroke** | **Number of molecules in the cylinder after the intake stroke** |
|---|---|---|---|
| Nitrogen | $1{,}647.52 \times 10^{19}$ | $183.06 \times 10^{19}$ | $1{,}830.58 \times 10^{19}$ |
| Oxygen | $441.98 \times 10^{19}$ | 0 | $441.98 \times 10^{19}$ |
| Octane | $35.36 \times 10^{19}$ | 0 | $35.36 \times 10^{19}$ |

| | | | |
|---|---|---|---|
| Argon | $19.71 \times 10^{19}$ | $2.19 \times 10^{19}$ | $21.90 \times 10^{19}$ |
| Carbon dioxide | $0.66 \times 10^{19}$ | $31.50 \times 10^{19}$ | $32.17 \times 10^{19}$ |
| Neon | $0.04 \times 10^{19}$ | $\approx 0$ | $0.04 \times 10^{19}$ |
| Helium | $0.01 \times 10^{19}$ | $\approx 0$ | $0.01 \times 10^{19}$ |
| Water vapour | $6.98 \times 10^{19}$ | $36.13 \times 10^{19}$ | $43.11 \times 10^{19}$ |
| Total | $2{,}152.26 \times 10^{19}$ | $252.89 \times 10^{19}$ | $2{,}405.16 \times 10^{19}$ |

Let's just summarize where we are. We have combined molecules from two separate sources to figure out the component gases in the cylinder at the end of the intake stroke. We are going to take these molecules through the rest of the power cycle. At the moment, we have $2{,}405.16 \times 10^{19}$ molecules in the cylinder. They occupy a volume of exactly 1.0 liter. The pressure on the cylinder walls is $96{,}259\ \mathrm{N/m^2}$ and the temperature is 16.74°C. As an aside, we can now verify our calculation of the weighted average temperature of the gases at the end of the intake stroke. We combined $2{,}152.26 \times 10^{19}$ molecules at a temperature of 288.15°K from the intake chamber and $252.89 \times 10^{19}$ molecules at a temperature of 304.73°K from the exhaust. The weighted average temperature is the following simple average: $[(2{,}152.26 \times 288.15) + (252.89 \times 304.73)]/(2{,}152.26 + 252.89) = 289.89$°K, or 16.74°C.

It happens that, in one of the steps below, we are also going to need to know the mass of the gas inside the cylinder. This is a good time to figure it out. www.EngineeringToolBox.com has tables of the molecular weights of the molecules we are looking at. The following table states the weight of each molecule in Atomic Mass Units. Since one AMU is equal to one gram per mole of any material, it is easy to calculate the mass of each gas in the cylinder at the end of the intake stroke. This is also shown in the table. The total mass of the gas in the cylinder is 0.0012044 kilograms, or 1.2044 grams.

| **Component gas** | **Number of molecules in the cylinder after the intake stroke** | **Atomic weight (AMU)** | **Mass in the cylinder after the intake stroke** |
|---|---|---|---|
| Nitrogen | $1{,}830.58 \times 10^{19}$ | 28.0134 | $8.5154 \times 10^{-4}$ kg |
| Oxygen | $441.98 \times 10^{19}$ | 31.9988 | $2.3485 \times 10^{-4}$ kg |
| Octane | $35.36 \times 10^{19}$ | 114.2200 | $6.7063 \times 10^{-5}$ kg |
| Argon | $21.90 \times 10^{19}$ | 39.9480 | $1.4525 \times 10^{-5}$ kg |
| Carbon dioxide | $32.17 \times 10^{19}$ | 44.0100 | $2.3507 \times 10^{-5}$ kg |
| Neon | $0.04 \times 10^{19}$ | 20.1790 | $1.4281 \times 10^{-8}$ kg |
| Helium | $0.01 \times 10^{19}$ | 4.0026 | $8.1649 \times 10^{-10}$ kg |
| Water vapour | $43.11 \times 10^{19}$ | 18.0200 | $1.2900 \times 10^{-5}$ kg |
| Total | $2{,}405.16 \times 10^{19}$ | | $1.2044 \times 10^{-3}$ kg |

**The compression stroke**

Let's look at the compression stroke next. It is the black line from Point B to Point C in the drawing. The trapped gas is squeezed from a volume of one liter into a smaller volume only one-tenth as big. The pressure rises, but it rises by more than a factor of ten. If the starting pressure is one atmosphere, or close to one atmosphere, the pressure when the piston reaches the top of the cylinder is going to be more that ten atmospheres. In our numerical example, it is about 21 atmospheres.

Squeezing a gas is a complicated physical process. We are going to model this complicated process as an "adiabatic process involving an ideal gas". That statement contains two important assumptions about what is going on. Firstly, the "ideal gas" assumption is that the molecules of the gas fly around like

mathematical points and bounce off each other (and the walls) in perfectly elastic collisions. To the extent that this assumption is valid, we can describe the properties of state using the Ideal Gas Law:

$$P \times V = n \times R \times T \quad - \text{ The Ideal Gas Law (1)}$$

The pressure, volume and temperature (measured in absolute degrees, or degrees Kelvin) are represented by the symbols $P$, $V$ and $T$. $n$ is the number of moles of gas trapped inside the cylinder and $R$ is the Ideal Gas Constant ($R = 8.314$ J/mole°K). Earlier papers describe this relationship in more detail.

The second assumption – the "adiabatic" assumption – is that compression takes place so quickly that there is no exchange of energy between the gas and its surroundings (the walls). The walls may be hot, but there is not enough time for most molecules to hit the walls and be speeded up, which would increase their energy. Similarly, although the gas is heating up during the compression, there is not enough time for most molecules to hit the walls and transfer some of their energy to the walls. In other words, the energy which the gas contains at the end of the intake stroke remains constant during the compression stroke. To the extent this assumption is valid[1], we can describe an adiabatic process using the relationship:

$$P \times V^{\gamma} = \text{constant} \quad - \text{ For an Adiabatic Process (2)}$$

In this equation, the symbol $\gamma$ (the Greek letter "gamma") is the Adiabtic Index of the gas. The Adiabatic Index accounts for the fact that the molecules hold their energy in two different forms. They have kinetic energy, which is related to the speed at which they fly around. They also have internal energy, which is related to the relative motion of the atoms inside the molecules. For the simplest kind of molecules, each having only one atom, $\gamma = 1.67$. Dry air, which is mostly $N_2$ and $O_2$, has $\gamma = 1.40$. As the complexity of the molecules increases, and the number of atoms per molecule increases, there become more ways for the atoms to move around. Since these molecules can store more energy internally, $\gamma$ decreases even more. $\gamma$ also changes a bit as temperature changes (except for monatomic molecules). This happens because there are different ways the constituent atoms can move with respect to each other and with respect to the center-of-gravity of the molecule. They can vibrate in simple modes or in more complicated modes, and they can rotate around different axes as well. Which modes of motion predominate depends on the temperature.

In our numerical example, we are going to use three values for $\gamma$, one for the compression stroke, another for the power stroke and a third during the exhaust stroke. Gasoline vapour, whose molecules are more complex than those of dry air, tend to reduce the $\gamma$ of the fuel-air mixture. A fuel-air mixture at atmospheric pressure has $\gamma = 1.35$. After being heated and compressed during the compression stroke, $\gamma$ will have been reduced, to $\gamma = 1.33$. After combustion, the molecules of the gas will have changed type. There will be much more carbon dioxide ($CO_2$) and water ($H_2O$), and much less (zero, if the burning was complete) oxygen ($O_2$). After combustion, the average molecule will be more complex than the diatomic molecules of nitrogen and oxygen which made up the most of the pre-combustion gas. $\gamma$ will have dropped even more, likely to the 1.20 to 1.25 range. The exhaust stroke is cooler than the combustion stroke, consistent with a slightly lower $\gamma$. We chose to use the following average values in the numerical example:

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[1] To be precise, use of Equation (2) depends on one additional assumption as well. The process must be "reversible". When a process is both adiabatic and reversible, it is said to be "isentropic". If there is friction during a process, it will not be reversible. We are not talking here about mechanical friction involving engine parts. We are talking about friction, for example, between atoms when one molecule hits another. For our purposes, this kind of friction is negligible.

| | |
|---|---|
| For the compression stroke | $\gamma = 1.340$ |
| For the power stroke | $\gamma = 1.225$ |
| For the exhaust stroke | $\gamma = 1.200$ |

Let us illustrate how to apply these two equations to the compression stroke. The state of the mixture at the end of the intake stroke, from above, is as follows:

At the start of the compression stroke:
$V_{start} = 1 \text{ liter} = 0.001 \text{ m}^3$
$P_{start} = 96{,}259 \text{ N/m}^2$
$n = 2{,}405.16 \times 10^{19} \text{ molecules} = 0.039939 \text{ moles}$
$T_{start} = 16.74°\text{C} = 289.89°\text{K}$

Assuming compression is adiabatic and reversible, the quantity $P \times V^{\gamma}$ will be constant throughout. At the start of the compression stroke, this value is:

$$P_{start} \times V_{start}^{\gamma} = 96{,}259 \times 0.001^{1.340} = 9.1926$$

This value must will also apply at the end of the compression stroke, when the volume will have been reduced to $V_{end} = V_{start}/10 = 0.0001\ m^3$. At that point:

$$P_{end} \times V_{end}^{\gamma} = 9.1926$$
$$\rightarrow \quad P_{end} = \frac{9.1926}{V_{end}^{\gamma}} = \frac{9.1926}{0.0001^{1.340}} = 2{,}105{,}900 \text{ N/m}^2 = 20.78 \text{ atmospheres}$$

In fact, one can make the same calculation for pressure at any volume along the way from BDC (one liter) to TDC (0.1 liter).

Next, at any point during the compression stroke for which we care to calculate the pressure, we can use the Ideal Gas Law to calculate the temperature as well. At the end of the compression stroke, for example, it is:

$$P_{end} \times V_{end} = n \times R \times T_{end}$$
$$\rightarrow \quad T_{end} = \frac{P_{end} \times V_{end}}{n \times R} = \frac{2{,}105{,}900 \times 0.0001}{0.039939 \times 8.314} = 634.22°\text{K} = 361.07°\text{C}$$

**Phase 1 of the power stroke - Combustion**

Now that the mixture has been compressed, we can fire the spark plug and begin the power stroke. This is also a complicated physical process. We are going to model it as taking place in two separate and sequential phases. These are shown separately in the pressure-versus-volume diagram above. There is the vertical line from Point C to Point D, followed by a curve towards the lower right from Point D to Point E. Here's the difference between the two phases. The first phase is the burning of the fuel-air mixture. We will assume that the burning takes place very quickly, so quickly that the piston does not even move while it takes place. Since the piston does not move, the volume is constant. That is why this part of the power stroke appears as a straight vertical line in the diagram. (A thermal process like this, which takes place at constant volume, is called an "isochoric" process.) Burning the fuel adds heat energy to the gas. The gas responds by expanding, driving the piston down. The expansion of the heated gas is

the second phase of the power stroke. We will describe that in the next section. In this section, we will look only at the burning phase.

Burning releases heat. The amount of heat released depends on the amount of octane and oxygen inside the cylinder. Experiments have shown that burning one kilogram of gasoline[2] releases 41.2 MJ (MegaJoules, or millions of Joules) of heat energy[3]. The most recent table above shows that the mass of octane inside the cylinder is $6.7063 \times 10^{-5}$ kg. Burning it will release $6.7063 \times 10^{-5} \times 41.2 = 0.002763$ MJ $= 2{,}763$ Joules of heat energy. Burning the fuel also changes the types of molecules inside the cylinder. There is enough information in the tables above for us to write down the molecules which are in the cylinder before and after the burning.

| Component gas | Number of molecules inside the cylinder | | |
|---|---|---|---|
| | **Before burning** | **Ratio to octane** | **After burning** |
| Nitrogen | $1{,}830.58 \times 10^{19}$ | | $1{,}830.58 \times 10^{19}$ |
| Oxygen | $441.98 \times 10^{19}$ | 12.5 : 1 | 0 |
| Octane | $35.36 \times 10^{19}$ | 1 : 1 | 0 |
| Argon | $21.90 \times 10^{19}$ | | $21.90 \times 10^{19}$ |
| Carbon dioxide | $32.17 \times 10^{19}$ | 8 : 1 | $315.03 \times 10^{19}$ |
| Neon | $0.04 \times 10^{19}$ | | $0.04 \times 10^{19}$ |
| Helium | $0.01 \times 10^{19}$ | | $0.01 \times 10^{19}$ |
| Water vapour | $43.11 \times 10^{19}$ | 9 : 1 | $361.34 \times 10^{19}$ |
| Total | $2{,}405.16 \times 10^{19}$ | | $2{,}528.91 \times 10^{19}$ |

Next, we have to figure out what the addition of this heat energy will do. It is a good assumption that the byproducts of the combustion still behave like an ideal gas. When an ideal gas is heated or cooled at constant volume, the governing relationship is:

$$\text{Heat absorbed} = \text{mass} \times C_V \times \text{Rise in temperature}$$

$$\rightarrow \qquad \text{Rise in temperature} = \frac{\text{Heat absorbed}}{\text{mass} \times C_V} \qquad (3)$$

where $C_V$ is the specific heat of the gas, measured at constant volume. We know the heat released by the burning ($2{,}763$ Joules), which is then absorbed by the gas. And, we know the total mass of the gas in the cylinder ($0.0012044$ kg), which doesn't change even though the burning reorganizes the atoms into different molecules.

In order to calculate the rise in temperature, we need to estimate the $C_V$ of the mixture of gases which is inside the cylinder. This is tricky for several reasons: (i) the specific heats of the individual gases are different, (ii) the specific heats change with temperature, and (iii) the types of gases do not remain constant while the burning takes place. In order to simplify things, we are going to make one more assumption. We will assume that the burning phase itself can be divided into two separate sub-phases.

---

[2] Our sources of data are a little bit in conflict. We set the stoichiometric ratio of octane to oxygen assuming the fuel is 100% octane, but the heat of combustion given here, taken from Wikipedia, is for "gasoline". We are going to live with this inconsistency because octane is by far the principal ingredient in gasoline.

[3] This value is the "lower" heat of combustion of gasoline. The "upper" heat of combustion is bigger, since it includes the additional energy which would be released if the products of the reaction were allowed to cool down and condense into liquids. That does not happen in an internal combustion engine.

In the first sub-phase, the heat energy released by combustion is absorbed by the gas. We will assume this is instantaneous. Then, in the second sub-phase, the temperature of the gas will increase as per Equation (3). This idealization eliminates reason (iii). The only gases we will need to take onto account when we estimate $C_V$ are those which exist after all the octane has been burned.

Tables of specific heat never give $C_V$. They give $C_P$ instead. $C_P$ is also a specific heat, but measured at constant pressure rather than at constant volume. Table-makers assume that users will convert from one to the other. For an ideal gas, the difference between them is the Ideal Gas Constant $R$, as in $C_P - C_V = R$. This relationship is closely connected to the definition of the Adiabatic Index, which we have encountered in the form $\gamma = C_P/C_V$. Table-makes usually give the value of $C_P$ in units of J/kg°K, that is, per kilogram of the gas. The Ideal Gas Constant which we have been using is $R = 8.314$ J/mole°K. Since $R$ is stated per mole of the gas, it can be used with any ideal gas whose quantity is measured in moles. Obviously, we can't subtract $R$ measured per mole from $C_P$ measured per kilogram. We have to convert one or the other.

Conversion is not hard, but must be done separately for each different gas under consideration. Let's look at an example. One molecule of carbon dioxide, for example, has a mass of 44.0100 AMU. One mole of carbon dioxide therefor has a mass of 0.4401 kg. Thus, the Ideal Gas Constant (to be used for carbon dioxide only) can be written as $R_{CO_2} = 8.314/0.4401 = 18.89$ J/kg°K. Tables of $C_P$ for carbon dioxide report that it is equal to 1,075 J/kg°K when the temperature is 600°K. Carbon dioxide's $C_V$ at this temperature is $C_V = C_P - R_{CO_2} = 1{,}075 - 18.89 = 1{,}056$ J/kg°K. This could be converted back into per-mole terms using the molecular weight, as $C_V = 1{,}056 \times 0.4401 = 464.7$ J/mole°K.

We will need to repeat this calculation for all six byproduct gases and at every temperature of interest. Rather than take space here in the text to show the results, we have placed them in Appendix "A. The following table copies from Appendix "A" the values of $C_V$ for the six gases at two particular temperatures whose significance we will explain in just a moment.

| **Temperature (°K)** | **Specific heat at constant volume, $C_V$, in J/kg-°K** | | | | | |
|---|---|---|---|---|---|---|
| | **Nitrogen gas** | **Argon** | **Carbon dioxide** | **Neon** | **Helium** | **Water vapour** |
| 650 | 789.2 | 314.9 | 913.1 | 618.0 | 3,110.9 | 1,585.6 |
| 2800 | 1,020.2 | 314.9 | 1,219.1 | 618.0 | 3,110.9 | 2,591.6 |

The denominator in Equation (3) involves the product of these $C_V$'s and the masses of their respective types. The table-before-last lists the molecules which are inside the cylinder. In the following table, these numbers of molecules are converted into moles and, then by multiplying by the molecular weights, into masses in kilograms.

| **Component gas** | **Gases inside the cylinder after burning** | | | |
|---|---|---|---|---|
| | **No. of molecules** | **No. of moles** | **Weight (AMU)** | **Mass (kg)** |
| Nitrogen | $1{,}830.58 \times 10^{19}$ | $3.0398\times 10^{-2}$ | 28.0134 | $8.5154\times 10^{-4}$ |
| Argon | $21.90 \times 10^{19}$ | $3.6360\times 10^{-4}$ | 39.9480 | $1.4525\times 10^{-5}$ |
| Carbon dioxide | $315.03 \times 10^{19}$ | $5.2313\times 10^{-3}$ | 44.0100 | $2.3023\times 10^{-4}$ |
| Neon | $0.04 \times 10^{19}$ | $7.0773\times 10^{-7}$ | 20.1790 | $1.4281\times 10^{-8}$ |
| Helium | $0.01 \times 10^{19}$ | $2.0399\times 10^{-7}$ | 4.0026 | $8.1649\times 10^{-10}$ |
| Water vapour | $361.34 \times 10^{19}$ | $6.0002\times 10^{-3}$ | 18.0200 | $1.0812\times 10^{-4}$ |
| Total | $2{,}528.91 \times 10^{19}$ | $4.1994\times 10^{-2}$ | | $1.2044\times 10^{-3}$ |

In the following table, we set out the multiplicative products – mass × $C_V$ – for the six gases and the two temperatures.

| Temperature (°K) | Mass × $C_V$ products, in J/°K | | | | | | |
|---|---|---|---|---|---|---|---|
| | Nitrogen gas | Argon | Carbon dioxide | Neon | Helium | Water vapour | Total |
| 650 | 0.672045 | 0.004574 | 0.210218 | 0.000009 | 0.000003 | 0.171442 | 1.058291 |
| 2800 | 0.868751 | 0.004574 | 0.280668 | 0.000009 | 0.000003 | 0.280214 | 1.434218 |

The two temperatures mentioned in this table look like they were selected at random. They weren't. We (the authors) have the advantage of already having completed the calculations. We (the authors) know that the temperature in the cylinder after compression, and just before the burning starts, is 634.22°K. We (the authors) also know the temperature at the end of the rise in temperature which is just about to start is approximately 2800°K. The two temperatures in our tables of specific heats which are closest to these beginning and ending temperatures are 650°K and 2800°K, respectively. The simple average of the mass × $C_V$ products for these two extremes is $(1.058291 + 1.434218) / 2 = 1.24625$ J/°K. This average value is the one we will substitute into Equation (3) as being representative of the entire process.

We have the data we need to use Equation (3). We get:

$$\text{Rise in temperature} = \frac{\text{Heat absorbed}}{\text{mass} \times C_V}$$

$$\rightarrow \quad \text{Rise in temperature} = \frac{2{,}763 \text{ J}}{1.24625 \text{ J/°K}} = 2{,}217\text{°K}$$

The temperature at the start of burning was $T_{start} = 634.22$°K. At the end of burning, it will be $T_{end} = 634.22 + 2{,}217 = 2{,}851$°K. The addition of heat to the gas raises its temperature. It raises the pressure in exactly the same proportion. Remember that we have assumed that the gas is still an ideal gas, so we can use the Ideal Gas Law to calculate the pressure at the end of this isochoric burning process:

$$P_{end} = \frac{n \times R \times T_{end}}{V_{end}}$$

Ah! A subtle issue arises. The number of moles of gas in the cylinder at the end of burning $n_{end}$ is not the same as the number of moles at the start. The composition of the molecules, and their number, has changed. Before burning, there were $n_{start} = 0.039939$ moles; after burning, there are $n_{end} = 0.041994$ moles.

$$P_{end} = \frac{n_{end} \times R \times T_{end}}{V_{end}} = \frac{0.041994 \times 8.314 \times 2{,}851}{0.0001} = 9{,}954{,}800 \text{ N/m}^2$$

**Phase 2 of the power stroke - Expansion**

In phase 1 of the power stroke, the fuel-air mixture was burned, the heat released was absorbed by the inert and byproduct gases, whose temperature rose. All of this was assumed to take place in such a short time that the piston head did not have a chance to move.

In phase 2 of the power stroke, the piston moves. It is forced downwards as the heated gas expands. The assumption we will make in order to model the behaviour of the gas is that this process is adiabatic and

reversible, which is to say, isentropic. This is the same assumption we made for the compression stroke. The justification for the assumption is the same as before, namely, that the expansion happens fast enough that the gas and the walls don't have enough time to exchange any meaningful amount of energy. The gas will have the same total amount of energy at the end of phase 2 as it had at the beginning. Equations (1) and (2) describe the state of the gas during phase 2, just like they did during the compression stroke. The principal difference is that the gas is at a much higher temperature than it was during the compression stroke. The second difference is that the types of gases inside the cylinder are now different, too. Both of these reasons cause the Adiabatic Index to be lower during expansion than they were during compression. We stated above that we would use an average value of $\gamma = 1.225$ for this process.

The state of the mixture at the end of the burning-induced rise in temperature was (from above):

At the start of the expansion phase of the power stroke:
$V_{start} = 0.1 \text{ liter} = 0.0001 \text{ m}^3$
$P_{start} = 9{,}954{,}800 \text{ N/m}^2$
$n = 2{,}528.91 \times 10^{19} \text{ molecules} = 0.041994 \text{ moles}$
$T_{start} = 2{,}851°\text{K}$

Assuming expansion is adiabatic and reversible, the quantity $P \times V^{\gamma}$ will be constant throughout. At the start of the expansion, its value is:

$$P_{start} \times V_{start}^{\gamma} = 9{,}954{,}800 \times 0.0001^{1.225} = 125.32$$

This must be its value at the end of the expansion, by which time the cylinder's volume will increased to one liter: $V_{end} = 0.001\ m^3$. At that moment:

$$P_{end} \times V_{end}^{\gamma} = 125.32$$
$$\rightarrow \quad P_{end} = \frac{125.32}{V_{end}^{\gamma}} = \frac{125.32}{0.001^{1.225}} = 592{,}967 \text{ N/m}^2 = 5.85 \text{ atmospheres}$$

For raphing purposes, one can make the same calculation of pressure at any volume along the way from TDC (0.1 liter) to BDC (1 liter).

At any time during the expansion for which we care to calculate the pressure, we can use the Ideal Gas Law to calculate the temperature as well. At the end of the expansion, for example, it is:

$$P_{end} \times V_{end} = n \times R \times T_{end}$$
$$\rightarrow \quad T_{end} = \frac{P_{end} \times V_{end}}{n \times R} = \frac{592{,}967 \times 0.001}{0.041994 \times 8.314} = 1{,}698°\text{K} = 1{,}425°\text{C}$$

**The exhaust stroke**

We discussed the exhaust stroke in some detail while we were sorting out the mixture of gases present during the intake stroke. As the piston head ascends with the exhaust valves open, 90% of the gas inside the cylinder will be forced out and into the exhaust manifold. (90% corresponds to the 10:1 compression ratio we are working with.)

Let's talk first about the 10% of the gas does not leave the cylinder. It remains inside and is mixed with the fresh fuel-air mixture drawn from the intake chamber. The diagram we presented above depicts how we handled it. We imagined that the part of the gas which does not leave the cylinder occupies one-tenth of the

cylinder before the exhaust stroke begins. At the end of the exhaust stroke, it still occupies that same volume. After all, it is the only gas left inside the cylinder at the end of the exhaust stroke.

Since the volume occupied by this residual gas does not change during the exhaust stroke, the process it undergoes is by definition an isochoric (constant volume) one. We can use the Ideal Gas Law to model the gas throughout the process. At the start of the exhaust stroke, the state of the residual gas is:

At the start of the exhaust stroke:

$V_{start} = 0.1 \text{ liter} = 0.0001 \text{ m}^3$

$P_{start} = 592{,}967 \text{ N/m}^2$

$n_{10\%} = 252.89 \times 10^{19} \text{ molecules} = 0.0041994 \text{ moles}$

$T_{start} = 1{,}698°\text{K}$

We know the pressure at the end of the exhaust stroke. Recall that we assumed at the outset that the exhaust gases would be forced out of the cylinder by a 5% overpressure. Assuming that the exhaust system has very little back-pressure, the pressure inside the cylinder at the end of the exhaust stroke will be 1.05 atmospheres, or $P_{end} = 1.05 \times 101{,}325 = 106{,}391 \text{ N/m}^2$ . Using the Ideal Gas Law to calculate the temperature gives:

$$T_{end} = \frac{P_{end} \times V_{end}}{n_{10\%} \times R} = \frac{106{,}391 \times 0.0001}{0.0041994 \times 8.314} = 304.73°\text{K} = 31.58°\text{C}$$

Note that this temperature (304.73°K) and this number of moles (0.0041994 moles) are the values we used in the section above dealing with the intake stroke. Several iterations of all these calculations were required to get things right, so the values we find here at the end of the calculations are the same as the ones we use when figuring out the intake stroke.

Now, let's talk about the 90% which leaves the cylinder during the exhaust stroke. When it was inside the cylinder, this escaping gas had a temperature of 1,698°K. As it passes through the exhaust valve, this gas will expand as it enters the greater volume of the exhaust manifold. Expansion lowers the temperature. The temperature decreases further as the exhaust gas mingles with cooler gas already in the exhaust system and then with the ambient air after it leaves the tailpipe. After all is said and done, the 90% which leaves the cylinder will have returned to ambient pressure $(101{,}325 \text{ N/m}^2)$ and temperature $(15°C = 288.15°K)$. We can use the Ideal Gas Law to calculate the volume of this exhausted gas after it has stabilized. Note that using the Ideal Gas Law for the ending conditions does not depend on the process through which the gas gets there. That it, it does not depend on whether the process is isochoric, or adiabatic, etc.

For the departing gas at the start of the exhaust stroke:

$V_{start} = 0.9 \text{ liter} = 0.0009 \text{ m}^3$

$P_{start} = 592{,}967 \text{ N/m}^2$

$n_{90\%} = 2{,}276.02 \times 10^{19} \text{ molecules} = 0.037794 \text{ moles}$

$T_{start} = 1{,}698°\text{K}$

For the departing gas after it is well–mixed with ambient air:

$P_{end} = 101{,}325 \text{ N/m}^2$

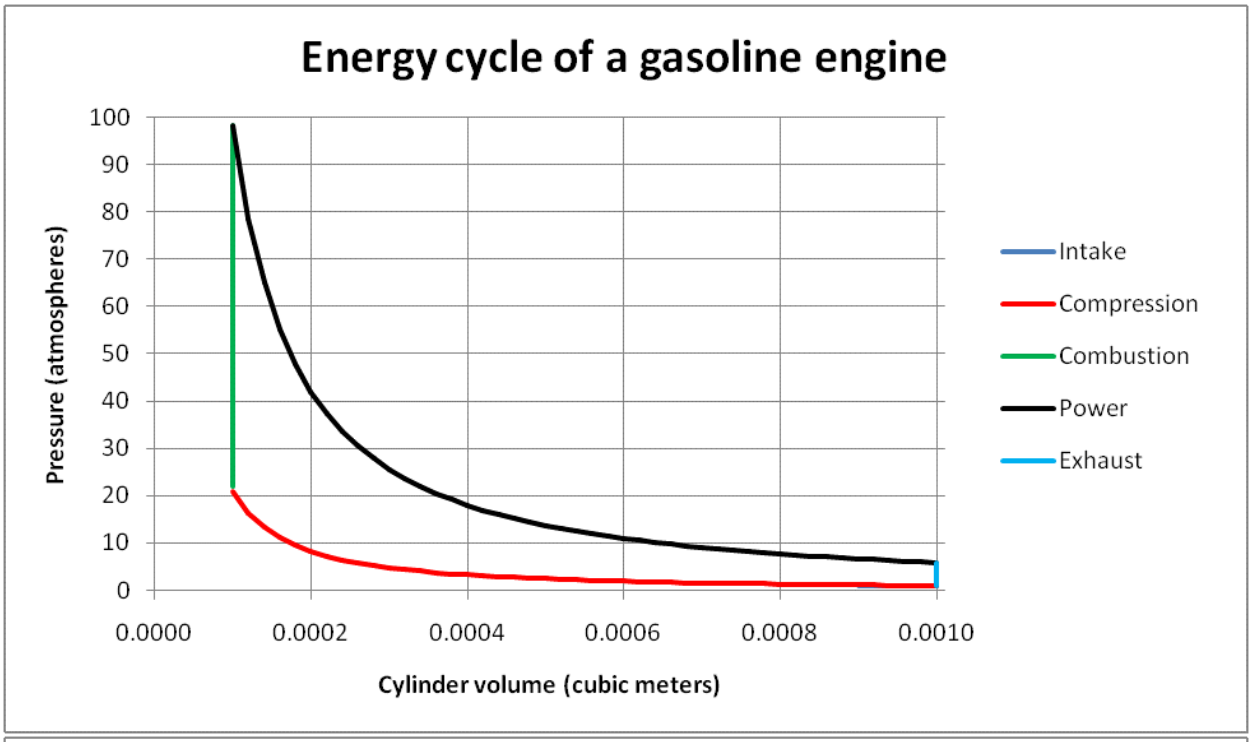
$n_{90\%} = 2{,}276.02 \times 10^{19} \text{ molecules} = 0.037794 \text{ moles}$

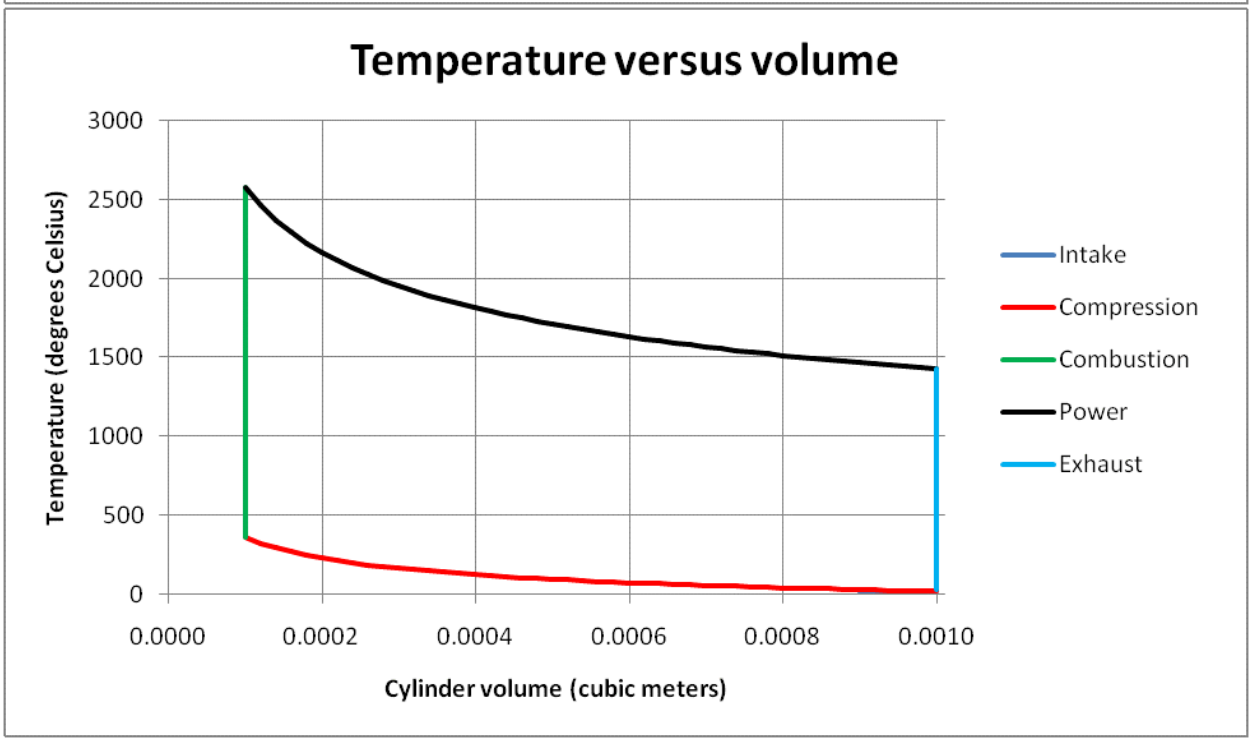
$T_{end} = 288.15°\text{K}$

$$V_{end} = \frac{n_{90\%} \times R \times T_{end}}{P_{end}} = \frac{0.037794 \times 8.314 \times 288.15}{101{,}325} = 0.0008936 \text{ m}^3$$

Note that, although the gas expands as it enters the exhaust manifold, it contracts as it mingles with cooler air. When all is said and done, its final volume (0.8936 liters) is just a little bit smaller than the volume it occupied inside the cylinder (0.9 liters).

We have now completed the calculation of the pressure, temperature and volume of the gas all the way through the power cycle or "all the way around" the cycle when plotted on a pressure-versus-volume or temperature-versus-volume map. The following two graphs show what we have calculated.



We now want to calculate the power produced during the cycle. We will then compare the power produced to the consumption of air. The place to start looking at power, which is the rate at which energy is produced, is to track the change in the energy of the gas through the various strokes of the cycle. You may not appreciate it yet, but we already have all the data we need, except for the exhaust stroke. Although we calculated the state of the gas (separately for the two parts of the combustion byproducts), we have not calculated how mush energy they lose. Let's do that now.

**Energy lost by the 10% of the exhaust gases which remain inside the cylinder**

This part of the gas stays at the same volume (0.1 liters) while its temperate decreases from 1,698°K to 304.7°K. Since this process takes place at constant volume ("isochoric"), we can use Equation (3) to calculate the heat energy lost. The heat lost is proportional to the decreases in temperature multiplied by $C_V$, the specific heat of the gas measured at constant volume. We did similar calculations when we examined the compression stroke above. The table of $C_V$ which we set up in Appendix "A" for that purpose can be used here, too. The following table copies from Appendix "A" the values of $C_V$ at the starting and ending temperatures which are closest to the ones which apply to the retained 10%.

| **Temperature (°K)** | **Specific heat at constant volume, $C_V$, in J/kg-°K** | | | | | |
|---|---|---|---|---|---|---|
| | **Nitrogen gas** | **Argon** | **Carbon dioxide** | **Neon** | **Helium** | **Water vapour** |
| 1700 | 966.2 | 314.9 | 1,159.1 | 618.0 | 3,110.9 | 2,249.6 |
| 300 | 743.2 | 314.9 | 657.1 | 618.0 | 3,110.9 | 1,402.6 |

The total mass of each gas inside the cylinder is exactly the same as it was after the burning, namely:

| | **Quantity of gas inside cylinder** | | | | | |
|---|---|---|---|---|---|---|
| | **Nitrogen gas** | **Argon** | **Carbon dioxide** | **Neon** | **Helium** | **Water vapour** |
| **Mass, kg** | $8.51538\times 10^{-4}$ | $1.45251\times 10^{-5}$ | $2.30228\times 10^{-4}$ | $1.42814\times 10^{-8}$ | $8.16488\times 10^{-10}$ | $1.08123\times 10^{-4}$ |

Multiplying these masses by their specific heats gives the following mass × $C_V$ produces:

| **Temp (°K)** | **Mass of the 10% part × $C_V$ products, in J/°K** | | | | | | |
|---|---|---|---|---|---|---|---|
| | **Nitrogen gas** | **Argon** | **Carbon dioxide** | **Neon** | **Helium** | **Water vapour** | **Total** |
| 1700 | 0.0822767 | 0.0004574 | 0.0266854 | 0.0000009 | 0.0000003 | 0.0243236 | 0.1337443 |
| 300 | 0.0632874 | 0.0004574 | 0.0151280 | 0.0000009 | 0.0000003 | 0.0151656 | 0.0940395 |

The simple average of the total mass × $C_V$ products at the two temperatures is $0.113892$ J/°K. The decrease in temperaure is $1{,}698°\text{K} - 304.73°\text{K} = 1{,}394°\text{K}$ and the energy released as the temperature falls is given by:

$$\begin{aligned}\text{Heat released} &= \text{mass} \times C_V \times \text{Fall in temperature} \\ &= 0.113892\ \text{J/°K} \times 1{,}394°\text{K} = 159\ \text{Joules}\end{aligned}$$

**Energy lost by the 90% of the exhaust gases which flow out the tailpipe**

This part of the gas stays at approximately the same volume while its temperaturee decreases from 1,698°K to 288.15°K. We would probably not be too far off if we approximated this entire process as being isochoric. However, to avoid leaving a needless inaccuracy, we will model the process undergone by this part of the gas in two separate and sequential phases. In the first phase, we will adjust the volume by the little bit needed to change the starting volume (0.9 liters) to the ending volume (0.8936 liters). We will model this small adjustment as being adiabatic. The advantage of using the adiabatic assumption is that it does not change the energy of the gas, so there will not be any heat release or absorption to calculate. Then, we will calculate the heat lost as the gas cools. Because the temperature in the cylinder is lower than it was during the burning, we will use a lower Adiabatic Index ($\gamma = 1.200$) than we used for the expansion which followed the burning. The calculations are as follows:

For the departing gas at the start of the exhaust stroke:

$V_{start} = 0.9 \text{ liter} = 0.0009 \text{ m}^3$

$P_{start} = 592{,}967 \text{ N/m}^2$

$n_{90\%} = 2{,}276.02 \times 10^{19} \text{ molecules} = 0.037794 \text{ moles}$

$T_{start} = 1{,}698°\text{K}$

The adiabatic relationship during the small adjustment:

$$P_{start} \times V_{start}^{\gamma} = P_{end} \times V_{end}^{\gamma}$$

$$\rightarrow P_{end} = \frac{P_{start} \times V_{start}^{\gamma}}{V_{end}^{\gamma}} = \frac{592{,}967 \times 0.0009^{1.200}}{0.0008936^{1.200}} = 598{,}077 \text{ N/m}^2$$

For the departing gas after the small adiabatic process to adjust volume:

$P_{end} = 598{,}077 \text{ N/m}^2$

$n_{90\%} = 2{,}276.02 \times 10^{19} \text{ molecules} = 0.037794 \text{ moles}$

$V_{end} = 0.0008936 \text{ m}^3$

$$T_{end} = \frac{P_{end} \times V_{end}}{n_{90\%} \times R} = \frac{598{,}077 \times 0.0008936}{0.037794 \times 8.314} = 1{,}701°\text{K}$$

The temperature of the departing gas falls from this temperature (1,701°K) to the ambient temperature (288.15°K) at a constant volume of $0.0008936 \text{ m}^3$. The fall in temperature is $1{,}701°\text{K} - 288.15°\text{K} = 1{,}413°\text{K}$. This is only a dozen or so degrees more than the fall in temperature experienced by the part of the gas which remains in the cylinder. As far as the right value for $C_V$ to use, the closest temperatures in the tables for $C_V$ contained in Appendix "A" are the same as for the 10%, being 1700°K and 300°K. Multiplying nine-tenths of the total mass inside the cylinder by the specific heats gives the following mass $\times$ $C_V$ products.

| Temp (°K) | Mass of the 90% part $\times$ $C_V$ products, in J/°K | | | | | | |
|---|---|---|---|---|---|---|---|
| | Nitrogen gas | Argon | Carbon dioxide | Neon | Helium | Water vapour | Total |
| 1700 | 0.7404907 | 0.0041163 | 0.2401689 | 0.0000079 | 0.0000023 | 0.2189124 | 1.2036985 |
| 300 | 0.5695870 | 0.0041163 | 0.1361520 | 0.0000079 | 0.0000023 | 0.1364902 | 0.8463558 |

The simple average of the total mass $\times$ $C_V$ products at the two temperatures is 1.025027 J/°K. The decrease in temperaure is 1,413°K so the energy lost as the temperature falls is given by:

$$\begin{aligned}\text{Heat released} &= \text{mass} \times C_V \times \text{Fall in temperature} \\ &= 1.025027\ \text{J/°K} \times 1{,}413\text{°K} = 1{,}448\ \text{Joules}\end{aligned}$$

**The energy balance for the whole cycle**

We have tried to follow the same molecules of gas all the way through one power cycle of the cylinder. Some of the changes to the gas were modeled as adiabatic (and reversible) processes. During those processes, the energy of the gas does not change. In fact, there were only two processes – the isochoric ones – during which the gas absorbed heat energy (during the burning phase of the power stroke) or lost heat energy (during the exhaust stroke). For the exhaust stroke, we modeled the gas in two parts, and both parts lost energy.

Since we have already calculated the energy gained or lost by the gas inside the cylinder during these isochoric processes, we can easily calculate the net thermal energy gained by the gas during one power cycle. The net thermal energy gained is:

$$\begin{aligned}\text{Net thermal energy gained by the gas} &= \begin{pmatrix}\text{Total thermal energy absorbed by the gas} \\ \text{less: Thermal energy lost by 10\% retained exhaust gas} \\ \text{less: Thermal energy lost by 90\% escaping exhaust gas}\end{pmatrix} \\ &= \begin{pmatrix}2{,}763\ \text{Joules} \\ -159\ \text{Joules} \\ -1{,}448\ \text{Joules}\end{pmatrix} \\ &= 1{,}156\ \text{Joules}\end{aligned}$$

Where did this energy go? A fundamental property of our universe is "conservation of energy". During any process, or series of processes, energy is not created and it does not disappear. All energy can do is change from one form to another. It is sometimes tricky to identify all the sources or uses of energy but, once they have been identified, it will be found that the total amount of energy in the "system" under consideration does not change.

In our case here, the net thermal energy gained by the gas is used up in several ways. Most of it is converted into mechanical energy, when the piston is forced to slide down the cylinder during the expansion phase of the power stroke. Some of that mechanical energy is lost as friction (which in due course becomes heat) as the piston, connecting rod and crankshaft slide against their supports. Some of the thermal energy is transferred to the mass of metal in the engine, which is in due course radiated away, or convected away, from the hot surface of the engine. What should we call these uses of the thermal energy? We will use the names "gross mechanical energy" and "heat radiated away from the engine". The "gross mechanical energy" is the mechanical energy excluding all of the effects of mechanical friction. The "heat radiated away from the engine" includes all three avenues by which the hot surface of the engine can dissipate heat. They are: (i) radiation of infrared rays, (ii) conduction through engine mounts and piping which touch the engine and (iii) convection directly into the air under the hood and surrounding the engine. The net thermal energy gained by the gas inside the cylinder is converted into the following forms of energy:

$$\begin{aligned}\text{Net thermal energy gained by the gas} &= \begin{pmatrix}\text{Gross mechanical energy} \\ \text{plus: Heat radiated away from the engine}\end{pmatrix} \\ \rightarrow \quad \text{Gross mechanical energy} &= \begin{pmatrix}\text{Net thermal energy gained by the gas} \\ \text{less: Heat radiated away from the engine}\end{pmatrix}\end{aligned}$$

As a rule of thumb, about 6% of the gross mechanical energy is lost to mechanical friction as it is conveyed from the piston head to the crankshaft. In this paper, we are going to focus on the power (or energy) available at the crankshaft, which we can therefore write as follows:

$$\text{Mechanical energy at the crankshaft} = 94\% \times \text{Gross mechanical energy}$$
$$= 94\% \times \begin{pmatrix} \text{Net thermal energy gained by the gas} \\ \text{less: Heat radiated away from the engine} \end{pmatrix}$$

Substituting the known value of the net thermal energy gained by the gas, we get:

$$\text{Mechanical energy at the crankshaft} = 94\% \times \begin{pmatrix} 1{,}156 \text{ Joules} \\ \text{less: Heat radiated away from the engine} \end{pmatrix}$$

Just to be clear, the 1,156 Joules substituted here is the net thermal energy we calculated for one power cycle of one cylinder whose nominal volume is one liter. It is worthwhile re-stating right here the amount of air which was used in that power cycle. We can pull down from above the number of molecules, and types of molecules, which were drawn into the cylinder during the intake stroke. We can multiply them by the atomic weight to calculate the total mass of the real air which was drawn in. We get:

| Component gas | Air drawn into cylinder during the intake stroke | | | |
|---|---|---|---|---|
| | **No. of molecules** | **No. of moles** | **AMU** | **Total mass (kg)** |
| Nitrogen | $1{,}647.52 \times 10^{19}$ | 0.0273578 | 28.0134 | $7.66384\times 10^{-4}$ |
| Oxygen | $441.98 \times 10^{19}$ | 0.0073393 | 31.9988 | $2.34848\times 10^{-4}$ |
| Argon | $19.71 \times 10^{19}$ | 0.0003272 | 39.9480 | $1.30726\times 10^{-5}$ |
| Carbon dioxide | $0.66 \times 10^{19}$ | 0.0000110 | 44.0100 | $4.84172\times 10^{-7}$ |
| Neon | $0.04 \times 10^{19}$ | 0.0000006 | 20.1790 | $1.28532\times 10^{-8}$ |
| Helium | $0.01 \times 10^{19}$ | 0.0000002 | 4.0026 | $7.34839\times 10^{-10}$ |
| Water vapour | $6.98 \times 10^{19}$ | 0.0001159 | 18.0200 | $2.08802\times 10^{-6}$ |
| Total | $2{,}152.26 \times 10^{19}$ | 0.0351520 | | $1.01689\times 10^{-3}$ |

So, the total mass of real air used is 0.00101689 kilograms.

The real air includes water vapour due to humidity in the air on the day of our test. Excluding the humidity, the mass of dry air drawn in is 0.00101480 kilograms.

There are several directions of analysis we can pursue. Let's look at the engine's power first.

**Engine power, ignoring the heat radiated away from the engine**

As a starting point, let's ignore entirely the heat radiated away from the engine. If we do that, then the mechanical energy available at the crankshaft (per power cycle of a one-liter cylinder) is equal to:

$$\text{Mechanical energy at the crankshaft (per cycle per liter)} = 94\% \times 1{,}156 \text{ Joules} = 1{,}087 \text{ Joules}$$

What if the engine in question is a 2.5-liter engine with four cylinders? Each cylinder will have a nominal volume of 2.5 / 4 = 0.625 liters. The mechanical energy at the crankshaft per cycle per cylinder will be equal to:

$$\text{Mechanical energy at the crankshaft (per cycle per cylinder)} = 0.625 \times 1{,}087 \text{ Joules}$$

Suppose the engine in question is turning over at 2,000 RPM. Since each power cycle takes two complete revolutions, and since there are four cylinders, the total number of power cycles in one minute will be equal to $1{,}000 \times 4 = 4{,}000$ and the total mechanical energy delivered to the crankshaft in one minute will be equal to:

$$\text{Total mechanical energy at the crackshaft (per minute)} = 4{,}000 \times 0.625 \times 1{,}087 \text{ Joules}$$

Dividing by 60 gives the total mechanical energy delivered to the crankshaft every second, thus

$$\text{Total mechanical energy at the crackshaft (per second)} = 4{,}000 \times 0.625 \times 1{,}087 \text{ Joules}/60$$

Power is the rate at which energy is produced or used. Since the previous expression is a measure of energy delivered per second, it is actually a statement of how much power is generated at the crankshaft. The metric (S.I.) unit of power is the Watt, and each Watt is one Joule per second. We have:

$$\text{Total power at the crankshaft} = 4{,}000 \times 0.625 \times 1{,}087 \,/60 = 45{,}290 \text{ Watts}$$

It is customary in automotive application to measure power in horsepower, and the conversion is done at the rate of $1 \text{ HP} = 745.7$ Watts. Applying this conversion factor gives:

$$\text{Total power at the crankshaft} = \frac{45{,}290}{745.7} = 60.7 \text{ HP}$$

Let's calculate the mass of real air drawn into the engine's four cylinders every minute. We will have to make some of the same adjustments as we did for the power. We will multiply the mass consumed by a one-liter cylinder by 0.625 to account for the smaller cylinders in this engine. We will multiply by 4,000 to account for the total number of power cycles in one minute. We get:

$$\text{Real air consumed per minute} = 4{,}000 \times 0.625 \times 0.00101689 = 2.5422 \text{ kilograms}$$

Kilograms are converted into pounds at the rate of 2.2 pounds per kilogram, so that:

$$\text{Real air consumed per minute} = 2.5422 \times 2.2 = 5.593 \text{ pounds}$$

The ratio of HP generated per pound of real air consumed per minute is:

$$\frac{\text{Total power at the crankshaft}}{\text{Real air consumed per minute}} = \frac{60.7}{5.593} = 10.85 \ \frac{\text{HP}}{\text{pound/minute}}$$

Repeating the calculation in terms of dry air, instead or real (humid) air, makes very little difference. The mass of dry air consumed per minute is $4{,}000 \times 0.625 \times 0.00101480 = 2.5370$ kilograms, or 5.581 pounds, and gives a power-to-consumption ratio of $60.7/5.581 = 10.88$ HP/pound/minute.

**Engine power, taking into account the heat radiated away from the engine**

Let's try to modify the calculation in the previous section to take into account the heat radiated away from the engine. So far as we (the authors) can tell, there is no rule-of-thumb for this loss. Obviously, it depends on the layout of the engine and the way it is cooled. Even so, we can make a plausible estimate.

A common household iron (for ironing clothes) is usually rated for 1,000 Watts. That is the electrical power it will consume when set on high. The purpose of the iron is to make heat, so virtually all of the electrical energy consumed is radiated away, or conducted away when the iron is pressed on a shirt. Our guess is that a typical engine running at 60 horsepower radiates as much heat as, say, five irons. That is equivalent to 5,000 Watts.

If 5,000 Watts is radiated away from the engine as heat, then only $45{,}290 - 5{,}000 = 40{,}290$ Watts of power will be generated at the crankshaft. This is equal to $40{,}290/745.7 = 54.0$ HP. The rate of consumption of air does not change, so the ratio of HP generated (at the crankshaft) per pound of real air consumed per minute will be reduced to:

$$\frac{\text{Total power at the crankshaft}}{\text{Real air consumed per minute}} = \frac{54.0}{5.593} = 9.65 \ \frac{\text{HP}}{\text{pound/minute}}$$

This is the figure we quoted at the outset of this paper as the result of our calculations.

**The flash point and the autoignition temperature**

The flash point of a combustible material is the lowest temperature at which it will ignite when prompted by an ignition source such as a spark plug. At temperatures near the flash point, the gas may even stop burning if the ignition source is removed. For gasoline, the flash point is said to be $-43°C$ (from Wikipedia). At this temperature, gasoline is a liquid.

The flash point should not be confused with the autoignition temperature. The autoignition temperature is the lowest temperature at which a gas mixture will ignite even in the absence of an ignition source. For gasoline, the autoignition temperature is said to be $280°C$ (again, from Wikipedia).

This presents us with a problem. In the text above, we calculated that the mixture inside the cylinder reaches a temperature of $361°C$ at the end of the compression stroke, even before the spark plug flash is intended to start the burning. This temperature is well above the autoignition temperature started above, at which the gasoline mixture should burst into flame even without the help of a spark plug. But, there are two reasons why autoignition is unlikely at the 10:1 compression ratio we have assumed. First, the autoignition temperature stated ($280°C$) assumes an ideal combination of gasoline vapour and oxygen. When the cylinder is only partly filled with gasoline vapour and oxygen, and is accompanied by a lot of other inert gases, the effective autoignition temperature is higher. Secondly, we modeled the compression stroke as a completely adiabatic process undertaken at a constant Adiabatic Index $\gamma$. In reality, the process is more complicated than that. A more sophisticated model would probably result in a lower temperature. Even so, autoignition is always a danger with spark-ignited gasoline fuel.

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We have attached as Appendix "B" a copy of the Excel spreadsheet we used to make our calculations and to prepare the graphs set out above.

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If you found our description helpful, please let us know. If you spot any errors or omissions, please send an e-mail. Thank you.

## Appendix "A"

### Calculation of the specific heat $C_V$ of the gases in the cylinder after combustion

The second column in the following table sets out the number of molecules inside the cylinder after the intake stroke, which will remain the same throughout the compression stroke. These figures are taken directly from one of the tables above in the section titled *The intake stroke*. The third column sets out the number of molecules inside the cylinder after the burning The figures in the third column are taken directly from another table, in the section titled *Phase 1 of the power stroke - Combustion.* The fourth column is a simple scaling of the third column by Avogadro's number, from molecules to moles.

| Component gas | Number of molecules in the cylinder after the intake stroke | Number of molecules in the cylinder after burning | Number of moles in the cylinder after burning |
|---|---|---|---|
| Nitrogen | $1{,}830.58 \times 10^{19}$ | $1{,}830.58 \times 10^{19}$ | $3.0398 \times 10^{-2}$ |
| Oxygen | $441.98 \times 10^{19}$ | 0 | 0 |
| Octane | $35.36 \times 10^{19}$ | 0 | 0 |
| Argon | $21.90 \times 10^{19}$ | $21.90 \times 10^{19}$ | $3.6360 \times 10^{-4}$ |
| Carbon dioxide | $32.17 \times 10^{19}$ | $315.03 \times 10^{19}$ | $5.2313 \times 10^{-3}$ |
| Neon | $0.04 \times 10^{19}$ | $0.04 \times 10^{19}$ | $7.0773 \times 10^{-7}$ |
| Helium | $0.01 \times 10^{19}$ | $0.01 \times 10^{19}$ | $2.0399 \times 10^{-7}$ |
| Water vapour | $43.11 \times 10^{19}$ | $361.34 \times 10^{19}$ | $6.0002 \times 10^{-3}$ |
| Total | $2{,}405.16 \times 10^{19}$ | $2{,}528.91 \times 10^{19}$ | $4.1994 \times 10^{-2}$ |

The website www.EngineeringToolBox.com gives tables of $C_P$ for these component gases over a wide range of temperatures. $C_P$ is the specific heat at constant pressure, which should not be confused with $C_V$, the specific heat at constant volume. Note that these figures for $C_P$ are stated in kiloJoules, not Joules.

| Temperature (°K) | Specific heat at constant pressure, $C_P$, in KJ/kg-°K | | | | | |
|---|---|---|---|---|---|---|
| | Nitrogen gas | Argon | Carbon dioxide | Neon | Helium | Water vapour |
| 175 | 1.039 | 0.523 | 0.709 | 1.030 | 5.188 | 1.850 |
| 200 | 1.039 | 0.523 | 0.735 | 1.030 | 5.188 | 1.851 |
| 225 | 1.039 | 0.523 | 0.763 | 1.030 | 5.188 | 1.852 |
| 250 | 1.039 | 0.523 | 0.791 | 1.030 | 5.188 | 1.855 |
| 275 | 1.039 | 0.523 | 0.819 | 1.030 | 5.188 | 1.859 |
| 300 | 1.040 | 0.523 | 0.846 | 1.030 | 5.188 | 1.864 |
| 325 | 1.040 | 0.523 | 0.871 | 1.030 | 5.188 | 1.871 |
| 350 | 1.041 | 0.523 | 0.895 | 1.030 | 5.188 | 1.880 |
| 375 | 1.042 | 0.523 | 0.918 | 1.030 | 5.188 | 1.890 |
| 400 | 1.044 | 0.523 | 0.939 | 1.030 | 5.188 | 1.901 |
| 450 | 1.049 | 0.523 | 0.978 | 1.030 | 5.188 | 1.926 |
| 500 | 1.056 | 0.523 | 1.014 | 1.030 | 5.188 | 1.954 |
| 550 | 1.065 | 0.523 | 1.046 | 1.030 | 5.188 | 1.984 |
| 600 | 1.075 | 0.523 | 1.075 | 1.030 | 5.188 | 2.015 |
| 650 | 1.086 | 0.523 | 1.102 | 1.030 | 5.188 | 2.047 |

| | | | | | | |
|---|---|---|---|---|---|---|
| 700 | 1.098 | 0.523 | 1.126 | 1.030 | 5.188 | 2.080 |
| 750 | 1.110 | 0.523 | 1.148 | 1.030 | 5.188 | 2.113 |
| 800 | 1.122 | 0.523 | 1.168 | 1.030 | 5.188 | 2.147 |
| 850 | 1.134 | 0.523 | 1.187 | 1.030 | 5.188 | 2.182 |
| 900 | 1.146 | 0.523 | 1.204 | 1.030 | 5.188 | 2.217 |
| 950 | 1.157 | 0.523 | 1.220 | 1.030 | 5.188 | 2.252 |
| 1000 | 1.167 | 0.523 | 1.234 | 1.030 | 5.188 | 2.288 |
| 1050 | 1.177 | 0.523 | 1.247 | 1.030 | 5.188 | 2.323 |
| 1100 | 1.187 | 0.523 | 1.259 | 1.030 | 5.188 | 2.358 |
| 1150 | 1.196 | 0.523 | 1.270 | 1.030 | 5.188 | 2.392 |
| 1200 | 1.204 | 0.523 | 1.280 | 1.030 | 5.188 | 2.425 |
| 1250 | 1.212 | 0.523 | 1.290 | 1.030 | 5.188 | 2.458 |
| 1300 | 1.219 | 0.523 | 1.298 | 1.030 | 5.188 | 2.490 |
| 1350 | 1.226 | 0.523 | 1.306 | 1.030 | 5.188 | 2.521 |
| 1400 | 1.232 | 0.523 | 1.313 | 1.030 | 5.188 | 2.552 |
| 1500 | 1.244 | 0.523 | 1.326 | 1.030 | 5.188 | 2.609 |
| 1600 | 1.254 | 0.523 | 1.338 | 1.030 | 5.188 | 2.662 |
| 1700 | 1.263 | 0.523 | 1.348 | 1.030 | 5.188 | 2.711 |
| 1800 | 1.271 | 0.523 | 1.356 | 1.030 | 5.188 | 2.756 |
| 1900 | 1.278 | 0.523 | 1.364 | 1.030 | 5.188 | 2.798 |
| 2000 | 1.284 | 0.523 | 1.371 | 1.030 | 5.188 | 2.836 |
| 2100 | 1.290 | 0.523 | 1.377 | 1.030 | 5.188 | 2.872 |
| 2200 | 1.295 | 0.523 | 1.383 | 1.030 | 5.188 | 2.904 |
| 2300 | 1.300 | 0.523 | 1.388 | 1.030 | 5.188 | 2.934 |
| 2400 | 1.304 | 0.523 | 1.393 | 1.030 | 5.188 | 2.962 |
| 2500 | 1.307 | 0.523 | 1.397 | 1.030 | 5.188 | 2.987 |
| 2600 | 1.311 | 0.523 | 1.401 | 1.030 | 5.188 | 3.011 |
| 2700 | 1.314 | 0.523 | 1.404 | 1.030 | 5.188 | 3.033 |
| 2800 | 1.317 | 0.523 | 1.408 | 1.030 | 5.188 | 3.053 |
| 2900 | 1.320 | 0.523 | 1.411 | 1.030 | 5.188 | 3.072 |
| 3000 | 1.323 | 0.523 | 1.414 | 1.030 | 5.188 | 3.090 |
| 3500 | 1.333 | 0.523 | 1.427 | 1.030 | 5.188 | 3.163 |
| 4000 | 1.342 | 0.523 | 1.437 | 1.030 | 5.188 | 3.217 |
| 4500 | 1.349 | 0.523 | 1.446 | 1.030 | 5.188 | 3.258 |
| 5000 | 1.355 | 0.523 | 1.455 | 1.030 | 5.188 | 3.292 |
| 5500 | 1.362 | 0.523 | 1.465 | 1.030 | 5.188 | 3.322 |
| 6000 | 1.369 | 0.523 | 1.476 | 1.030 | 5.188 | 3.350 |

The universal Ideal Gas Constant $R$ is converted into gas-specific Gas Constants as follows. The gas-specific Gas Constants in this table have units of J/kg°K.

| | Component gas | | | | | |
|---|---|---|---|---|---|---|
| | **Nitrogen gas** | **Argon** | **Carbon dioxide** | **Neon** | **Helium** | **Water vapour** |
| Mass (AMU) | 28.01340 | 39.94800 | 44.01000 | 20.17900 | 4.00260 | 18.02000 |
| Mass (kg/mole) | 0.028013 | 0.039948 | 0.044010 | 0.020179 | 0.004000 | 0.018020 |
| $R_{specific}$ | 296.79 | 208.12 | 188.91 | 412.01 | 2077.15 | 461.38 |

When we subtract $R_{specific}$ from the $C_P$'s in the table before last, we get the following values for $C_V$, in units of J/kg°K.

| **Temperature (°K)** | **Specific heat at constant volume, $C_V$, in J/kg-°K** | | | | | |
|---|---|---|---|---|---|---|
| | **Nitrogen gas** | **Argon** | **Carbon dioxide** | **Neon** | **Helium** | **Water vapour** |
| 175 | 742.2 | 314.9 | 520.1 | 618.0 | 3,110.9 | 1,388.6 |
| 200 | 742.2 | 314.9 | 546.1 | 618.0 | 3,110.9 | 1,389.6 |
| 225 | 742.2 | 314.9 | 574.1 | 618.0 | 3,110.9 | 1,390.6 |
| 250 | 742.2 | 314.9 | 602.1 | 618.0 | 3,110.9 | 1,393.6 |
| 275 | 742.2 | 314.9 | 630.1 | 618.0 | 3,110.9 | 1,397.6 |
| 300 | 743.2 | 314.9 | 657.1 | 618.0 | 3,110.9 | 1,402.6 |
| 325 | 743.2 | 314.9 | 682.1 | 618.0 | 3,110.9 | 1,409.6 |
| 350 | 744.2 | 314.9 | 706.1 | 618.0 | 3,110.9 | 1,418.6 |
| 375 | 745.2 | 314.9 | 729.1 | 618.0 | 3,110.9 | 1,428.6 |
| 400 | 747.2 | 314.9 | 750.1 | 618.0 | 3,110.9 | 1,439.6 |
| 450 | 752.2 | 314.9 | 789.1 | 618.0 | 3,110.9 | 1,464.6 |
| 500 | 759.2 | 314.9 | 825.1 | 618.0 | 3,110.9 | 1,492.6 |
| 550 | 768.2 | 314.9 | 857.1 | 618.0 | 3,110.9 | 1,522.6 |
| 600 | 778.2 | 314.9 | 886.1 | 618.0 | 3,110.9 | 1,553.6 |
| 650 | 789.2 | 314.9 | 913.1 | 618.0 | 3,110.9 | 1,585.6 |
| 700 | 801.2 | 314.9 | 937.1 | 618.0 | 3,110.9 | 1,618.6 |
| 750 | 813.2 | 314.9 | 959.1 | 618.0 | 3,110.9 | 1,651.6 |
| 800 | 825.2 | 314.9 | 979.1 | 618.0 | 3,110.9 | 1,685.6 |
| 850 | 837.2 | 314.9 | 998.1 | 618.0 | 3,110.9 | 1,720.6 |
| 900 | 849.2 | 314.9 | 1,015.1 | 618.0 | 3,110.9 | 1,755.6 |
| 950 | 860.2 | 314.9 | 1,031.1 | 618.0 | 3,110.9 | 1,790.6 |
| 1000 | 870.2 | 314.9 | 1,045.1 | 618.0 | 3,110.9 | 1,826.6 |
| 1050 | 880.2 | 314.9 | 1,058.1 | 618.0 | 3,110.9 | 1,861.6 |
| 1100 | 890.2 | 314.9 | 1,070.1 | 618.0 | 3,110.9 | 1,896.6 |
| 1150 | 899.2 | 314.9 | 1,081.1 | 618.0 | 3,110.9 | 1,930.6 |
| 1200 | 907.2 | 314.9 | 1,091.1 | 618.0 | 3,110.9 | 1,963.6 |
| 1250 | 915.2 | 314.9 | 1,101.1 | 618.0 | 3,110.9 | 1,996.6 |
| 1300 | 922.2 | 314.9 | 1,109.1 | 618.0 | 3,110.9 | 2,028.6 |
| 1350 | 929.2 | 314.9 | 1,117.1 | 618.0 | 3,110.9 | 2,059.6 |

| | | | | | | |
|---|---|---|---|---|---|---|
| 1400 | 935.2 | 314.9 | 1,124.1 | 618.0 | 3,110.9 | 2,090.6 |
| 1500 | 947.2 | 314.9 | 1,137.1 | 618.0 | 3,110.9 | 2,147.6 |
| 1600 | 957.2 | 314.9 | 1,149.1 | 618.0 | 3,110.9 | 2,200.6 |
| 1700 | 966.2 | 314.9 | 1,159.1 | 618.0 | 3,110.9 | 2,249.6 |
| 1800 | 974.2 | 314.9 | 1,167.1 | 618.0 | 3,110.9 | 2,294.6 |
| 1900 | 981.2 | 314.9 | 1,175.1 | 618.0 | 3,110.9 | 2,336.6 |
| 2000 | 987.2 | 314.9 | 1,182.1 | 618.0 | 3,110.9 | 2,374.6 |
| 2100 | 993.2 | 314.9 | 1,188.1 | 618.0 | 3,110.9 | 2,410.6 |
| 2200 | 998.2 | 314.9 | 1,194.1 | 618.0 | 3,110.9 | 2,442.6 |
| 2300 | 1,003.2 | 314.9 | 1,199.1 | 618.0 | 3,110.9 | 2,472.6 |
| 2400 | 1,007.2 | 314.9 | 1,204.1 | 618.0 | 3,110.9 | 2,500.6 |
| 2500 | 1,010.2 | 314.9 | 1,208.1 | 618.0 | 3,110.9 | 2,525.6 |
| 2600 | 1,014.2 | 314.9 | 1,212.1 | 618.0 | 3,110.9 | 2,549.6 |
| 2700 | 1,017.2 | 314.9 | 1,215.1 | 618.0 | 3,110.9 | 2,571.6 |
| 2800 | 1,020.2 | 314.9 | 1,219.1 | 618.0 | 3,110.9 | 2,591.6 |
| 2900 | 1,023.2 | 314.9 | 1,222.1 | 618.0 | 3,110.9 | 2,610.6 |
| 3000 | 1,026.2 | 314.9 | 1,225.1 | 618.0 | 3,110.9 | 2,628.6 |
| 3500 | 1,036.2 | 314.9 | 1,238.1 | 618.0 | 3,110.9 | 2,701.6 |
| 4000 | 1,045.2 | 314.9 | 1,248.1 | 618.0 | 3,110.9 | 2,755.6 |
| 4500 | 1,052.2 | 314.9 | 1,257.1 | 618.0 | 3,110.9 | 2,796.6 |
| 5000 | 1,058.2 | 314.9 | 1,266.1 | 618.0 | 3,110.9 | 2,830.6 |
| 5500 | 1,065.2 | 314.9 | 1,276.1 | 618.0 | 3,110.9 | 2,860.6 |
| 6000 | 1,072.2 | 314.9 | 1,287.1 | 618.0 | 3,110.9 | 2,888.6 |

Despite the title of this appendix, which says we are only going to calculate the $C_V$'s, we are going to go one step further. We know how much mass of each gas in present in the cylinder after the burning. We are going to multiply every column in the previous table by the mass of that gas so that we get products of mass $\times C_V$. We can add them up linearly, to get the mass $\times C_V$ product for all the gases inside the cylinder. Since each component gas can be treated as an ideal gas, they will absorb heat independently from each other at the rates given, so long as they all share the same temperature as it rises.

The mass of each gas in the cylinder (taken from a table in the text) is as follows:

| | **Mass of gas inside cylinder after burning** | | | | | |
|---|---|---|---|---|---|---|
| | **Nitrogen gas** | **Argon** | **Carbon dioxide** | **Neon** | **Helium** | **Water vapour** |
| **Mass (kg)** | $8.51538 \times 10^{-4}$ | $1.45251 \times 10^{-5}$ | $2.30228 \times 10^{-4}$ | $1.42814 \times 10^{-8}$ | $8.16488 \times 10^{-10}$ | $1.08123 \times 10^{-4}$ |

By multiplying the masses and the $C_V$'s, we get the following table.

| Temperature (°K) | Mass × $C_V$ products, in J/ °K | | | | | | |
|---|---|---|---|---|---|---|---|
| | Nitrogen gas | Argon | Carbon dioxide | Neon | Helium | Water vapour | Total |
| 175 | 0.632023 | 0.004574 | 0.119739 | 0.000009 | 0.000003 | 0.150142 | 0.906489 |
| 200 | 0.632023 | 0.004574 | 0.125725 | 0.000009 | 0.000003 | 0.150250 | 0.912583 |
| 225 | 0.632023 | 0.004574 | 0.132171 | 0.000009 | 0.000003 | 0.150358 | 0.919137 |
| 250 | 0.632023 | 0.004574 | 0.138617 | 0.000009 | 0.000003 | 0.150683 | 0.925908 |
| 275 | 0.632023 | 0.004574 | 0.145064 | 0.000009 | 0.000003 | 0.151115 | 0.932787 |
| 300 | 0.632874 | 0.004574 | 0.151280 | 0.000009 | 0.000003 | 0.151656 | 0.940395 |
| 325 | 0.632874 | 0.004574 | 0.157036 | 0.000009 | 0.000003 | 0.152413 | 0.946908 |
| 350 | 0.633726 | 0.004574 | 0.162561 | 0.000009 | 0.000003 | 0.153386 | 0.954258 |
| 375 | 0.634578 | 0.004574 | 0.167856 | 0.000009 | 0.000003 | 0.154467 | 0.961486 |
| 400 | 0.636281 | 0.004574 | 0.172691 | 0.000009 | 0.000003 | 0.155656 | 0.969213 |
| 450 | 0.640538 | 0.004574 | 0.181670 | 0.000009 | 0.000003 | 0.158359 | 0.985153 |
| 500 | 0.646499 | 0.004574 | 0.189958 | 0.000009 | 0.000003 | 0.161387 | 1.002429 |
| 550 | 0.654163 | 0.004574 | 0.197326 | 0.000009 | 0.000003 | 0.164631 | 1.020704 |
| 600 | 0.662678 | 0.004574 | 0.204002 | 0.000009 | 0.000003 | 0.167982 | 1.039248 |
| 650 | 0.672045 | 0.004574 | 0.210218 | 0.000009 | 0.000003 | 0.171442 | 1.058291 |
| 700 | 0.682264 | 0.004574 | 0.215744 | 0.000009 | 0.000003 | 0.175010 | 1.077603 |
| 750 | 0.692482 | 0.004574 | 0.220809 | 0.000009 | 0.000003 | 0.178578 | 1.096454 |
| 800 | 0.702701 | 0.004574 | 0.225413 | 0.000009 | 0.000003 | 0.182255 | 1.114954 |
| 850 | 0.712919 | 0.004574 | 0.229788 | 0.000009 | 0.000003 | 0.186039 | 1.133331 |
| 900 | 0.723138 | 0.004574 | 0.233702 | 0.000009 | 0.000003 | 0.189823 | 1.151247 |
| 950 | 0.732504 | 0.004574 | 0.237385 | 0.000009 | 0.000003 | 0.193608 | 1.168082 |
| 1000 | 0.741020 | 0.004574 | 0.240608 | 0.000009 | 0.000003 | 0.197500 | 1.183713 |
| 1050 | 0.749535 | 0.004574 | 0.243601 | 0.000009 | 0.000003 | 0.201284 | 1.199006 |
| 1100 | 0.758051 | 0.004574 | 0.246364 | 0.000009 | 0.000003 | 0.205069 | 1.214068 |
| 1150 | 0.765714 | 0.004574 | 0.248897 | 0.000009 | 0.000003 | 0.208745 | 1.227941 |
| 1200 | 0.772527 | 0.004574 | 0.251199 | 0.000009 | 0.000003 | 0.212313 | 1.240623 |
| 1250 | 0.779339 | 0.004574 | 0.253501 | 0.000009 | 0.000003 | 0.215881 | 1.253306 |
| 1300 | 0.785300 | 0.004574 | 0.255343 | 0.000009 | 0.000003 | 0.219341 | 1.264569 |
| 1350 | 0.791261 | 0.004574 | 0.257185 | 0.000009 | 0.000003 | 0.222693 | 1.275723 |
| 1400 | 0.796370 | 0.004574 | 0.258796 | 0.000009 | 0.000003 | 0.226044 | 1.285796 |
| 1500 | 0.806588 | 0.004574 | 0.261789 | 0.000009 | 0.000003 | 0.232207 | 1.305170 |
| 1600 | 0.815104 | 0.004574 | 0.264552 | 0.000009 | 0.000003 | 0.237938 | 1.322179 |
| 1700 | 0.822767 | 0.004574 | 0.266854 | 0.000009 | 0.000003 | 0.243236 | 1.337443 |
| 1800 | 0.829580 | 0.004574 | 0.268696 | 0.000009 | 0.000003 | 0.248102 | 1.350962 |
| 1900 | 0.835541 | 0.004574 | 0.270538 | 0.000009 | 0.000003 | 0.252643 | 1.363306 |
| 2000 | 0.840650 | 0.004574 | 0.272150 | 0.000009 | 0.000003 | 0.256751 | 1.374136 |
| 2100 | 0.845759 | 0.004574 | 0.273531 | 0.000009 | 0.000003 | 0.260644 | 1.384519 |
| 2200 | 0.850017 | 0.004574 | 0.274912 | 0.000009 | 0.000003 | 0.264104 | 1.393618 |
| 2300 | 0.854274 | 0.004574 | 0.276063 | 0.000009 | 0.000003 | 0.267347 | 1.402270 |

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| 2400 | 0.857681 | 0.004574 | 0.277215 | 0.000009 | 0.000003 | 0.270375 | 1.409855 |
| 2500 | 0.860235 | 0.004574 | 0.278136 | 0.000009 | 0.000003 | 0.273078 | 1.416034 |
| 2600 | 0.863641 | 0.004574 | 0.279056 | 0.000009 | 0.000003 | 0.275673 | 1.422956 |
| 2700 | 0.866196 | 0.004574 | 0.279747 | 0.000009 | 0.000003 | 0.278052 | 1.428580 |
| 2800 | 0.868751 | 0.004574 | 0.280668 | 0.000009 | 0.000003 | 0.280214 | 1.434218 |
| 2900 | 0.871305 | 0.004574 | 0.281359 | 0.000009 | 0.000003 | 0.282268 | 1.439517 |
| 3000 | 0.873860 | 0.004574 | 0.282049 | 0.000009 | 0.000003 | 0.284215 | 1.444709 |
| 3500 | 0.882375 | 0.004574 | 0.285042 | 0.000009 | 0.000003 | 0.292108 | 1.464110 |
| 4000 | 0.890039 | 0.004574 | 0.287345 | 0.000009 | 0.000003 | 0.297946 | 1.479915 |
| 4500 | 0.896000 | 0.004574 | 0.289417 | 0.000009 | 0.000003 | 0.302379 | 1.492381 |
| 5000 | 0.901109 | 0.004574 | 0.291489 | 0.000009 | 0.000003 | 0.306055 | 1.503238 |
| 5500 | 0.907070 | 0.004574 | 0.293791 | 0.000009 | 0.000003 | 0.309299 | 1.514745 |
| 6000 | 0.913030 | 0.004574 | 0.296324 | 0.000009 | 0.000003 | 0.312327 | 1.526266 |

Note that the values in this table are the $\text{mass} \times C_V$ products for 100% of the gas inside the cylinder after burning. If one is looking at the 10% part or the 90% part of the exhaust gases, then these values need to be scaled down to 10% or 90% of the total, as the case may be.

## Appendix "B"

## Excel spreadsheet

**Energy_cycle_of_a_gasoline_engine**
**(User inputs are highlighted in green)**
**(Constants are highlighed in pink)**

| | | | |
|---|---|---|---|
| Fundamental physical constants: | | | |
| R (Ideal Gas Constant) | | 8.314 | J/mole-degK |
| Temperature scales | 0 degC = | 273.15 | degK |
| Avogadro's number | 1 mole= | 6.0221413E+23 | molecules |
| Atomic weight | 1 AMU= | 1.6605389E-27 | kg |
| Horsepower/Watt | 1 Watt= | 745.699872 | HP |
| | | | |
| Engine parameters: | | | |
| Compression ratio | | 10 | (Range is 9 - 10.5 for gasoline engines) |
| Cylinder volume (max) | | 0.0010000 | m^3 |
| Cylinder volume (min) | | 0.0001000 | m^3 |
| Swept volume | | 0.0009000 | |
| Intake underpressure | | 5.00% | |
| Exhaust overpressure | | 5.00% | |
| | | | |
| Ambient air pressure and temperature: | | | |
| Pressure | 1 atm= | 101,325.00 | N/m^2 |
| Ambient temperature (degC) | | 15.00 | degC |
| Ambient temperature (degK) | | 288.15 | degK |
| | | | |
| Humidity: | | | |
| Saturation vapour pressure | | 1,670.00 | N/m^2 |
| Relative humidity | | 20.00% | |
| Actual vapour pressure | | 334.00 | N/m^2 |

Preliminary calculations relating to the ambient air

| Component gas | Partial pressure (N/m^2) | Number of moles per cubic meter = PV / RT | Number of molecules per cubic meter |
|---|---|---|---|
| Clean dry air | 100,991.00 | 42.155479 | 2.538663E+25 |
| Water vapour | 334.00 | 0.139418 | 8.395929E+22 |
| Total for real air | 101,325.00 | 42.294897 | 2.547058E+25 |

| Component gas | Fraction by volume | Number of molecules per cubic meter |
|---|---|---|
| Nitrogen (N2) | 78.084000% | 1.982298E+25 |
| Oxygen (O2) | 20.947600% | 5.317913E+24 |
| Argon (Ar) | 0.934000% | 2.371122E+23 |
| Carbon dioxide (CO2) | 0.031400% | 7.971437E+21 |
| Neon (Ne) | 0.001818% | 4.615310E+20 |
| Helium (He) | 0.000524% | 1.330265E+20 |
| Methane (CH4) | 0.000200% | 5.077348E+19 |
| Total for clean dry air | 99.999542% | 2.538663E+25 |
| Water vapour | | 8.395929E+22 |
| Total for real air | | 2.547058E+25 |

| | Per Octane molecule | Total molecules |
|---|---|---|
| Number of octane molecules | | 4.254330E+23 |
| Number of oxygen molecules = | 12.5 | 5.317913E+24 |
| Number of water vapour molecules = | 9.0 | 3.828897E+24 |
| Number of carbon dioxide molecules = | 8.0 | 3.403464E+24 |

| | Volume is slighly more than one cubic meter | | | |
|---|---|---|---|---|
| | Before combustion | | After combustion | |
| Component gas | Molecules | Percentage | Molecules | Percentage |
| Nitrogen | 1.982298E+25 | 76.548385% | 1.982298E+25 | 72.386194% |
| Oxygen | 5.317913E+24 | 20.535640% | 0 | 0.000000% |
| Octane | 4.254330E+23 | 1.642851% | 0 | 0.000000% |
| Argon | 2.371122E+23 | 0.915632% | 2.371122E+23 | 0.865846% |
| Carbon dioxide | 7.971437E+21 | 0.030782% | 3.411436E+24 | 12.457300% |
| Neon | 4.615310E+20 | 0.001782% | 4.615310E+20 | 0.001685% |
| Helium | 1.330265E+20 | 0.000514% | 1.330265E+20 | 0.000486% |
| Methane | 5.077348E+19 | 0.000196% | 5.077348E+19 | 0.000185% |
| Water vapour | 8.395929E+22 | 0.324217% | 3.912857E+24 | 14.288304% |
| Total for fuel-air mixture | 2.589602E+25 | 100.000000% | 2.738503E+25 | 100.000000% |

| Per Octane molecule | Total molecules |
|---|---|
| | 4.184438E+23 |
| 12.5 | 5.230548E+24 |
| 9.0 | 3.765994E+24 |
| 8.0 | 3.347550E+24 |

| | Re-scaled to a volume of exactly one cubic meter | | | |
|---|---|---|---|---|
| | Before combustion | | After combustion | |
| Comp. gas | Molecules | Percentage | Molecules | Percentage |
| Nitrogen | 1.949732E+25 | 76.548385% | 1.949732E+25 | 72.386194% |
| Oxygen | 5.230548E+24 | 20.535640% | 0 | 0.000000% |
| Octane | 4.184438E+23 | 1.642851% | 0 | 0.000000% |
| Argon | 2.332168E+23 | 0.915632% | 2.332168E+23 | 0.865846% |
| Carbon Diox. | 7.840478E+21 | 0.030782% | 3.355391E+24 | 12.457300% |
| Neon | 4.539487E+20 | 0.001782% | 4.539487E+20 | 0.001685% |
| Helium | 1.308411E+20 | 0.000514% | 1.308411E+20 | 0.000486% |
| Methane | 4.993935E+19 | 0.000196% | 4.993935E+19 | 0.000185% |
| Water vapour | 8.257997E+22 | 0.324217% | 3.848574E+24 | 14.288304% |
| Total | 2.547058E+25 | 100.000000% | 2.693514E+25 | 100.000000% |

## The intake stroke

Gas left over from the previous exhaust stroke

| | | |
|---|---|---|
| Volume (m^3) | 0.0001000 | = Unswept volume |
| Pressure (N/m^2) | 106,391.25 | = Ambient pressure plus exhaust overpressure |
| Temperature (degK) | 304.73 | <----- Copied up from the end of the exhaust stroke |
| Number of moles | 0.004199354 | <----- Copied up from the beginning of the exhaust stroke |
| Number of molecules | 2.528910E+21 | Define as Quantity "B" |

Total gas at the end of the intake stroke

| | | |
|---|---|---|
| Volume (m^3) | 0.0010000 | = Total cylinder volume |
| Pressure (N/m^2) | 96,258.75 | = Ambient pressure less intake underpressure |
| Temperature (degK) | 289.89 | <----- Linear average of temperature x molecules |
| Number of moles | 0.039938541 | Calculated using the Ideal Gas Law |
| Number of molecules | 2.405155E+22 | Define as Quantity "A" |

Gas drawn in during the intake stroke

| | | |
|---|---|---|
| Number of molecules | 2.152264E+22 | = Quantity "A" less Quantity "B" |
| Temperature (degK) | 288.15 | |

| Component gas | Number of new molecules drawn in | Number of molecules from exhaust | Number of molecules in cylinder | Molecular weight (AMU) | Total mass (kg) |
|---|---|---|---|---|---|
| Nitrogen | 1.647524E+22 | 1.830582E+21 | 1.830582E+22 | 28.0134 | 8.515380E-04 |
| Oxygen | 4.419813E+21 | 0 | 4.419813E+21 | 31.9988 | 2.348479E-04 |
| Octane | 3.535850E+20 | 0 | 3.535850E+20 | 114.2200 | 6.706332E-05 |
| Argon | 1.970682E+20 | 2.189646E+19 | 2.189646E+20 | 39.9480 | 1.452506E-05 |
| Carbon dioxide | 6.625204E+18 | 3.150339E+20 | 3.216591E+20 | 44.0100 | 2.350695E-05 |
| Neon | 3.835866E+17 | 4.262074E+16 | 4.262074E+17 | 20.1790 | 1.428136E-08 |
| Helium | 1.105607E+17 | 1.228453E+16 | 1.228453E+17 | 4.0026 | 8.164877E-10 |
| Methane | 4.219875E+16 | 4.688750E+15 | 4.688750E+16 | 16.0440 | 1.249162E-09 |
| Water vapour | 6.978007E+19 | 3.613384E+20 | 4.311184E+20 | 18.0200 | 1.290032E-05 |
| Total for fuel-air mixture | 2.152264E+22 | 2.528910E+21 | 2.405155E+22 | | 1.204398E-03 |

| For graphing purposes only | Pressure (N/m^2) | Pressure (atm) | Volume (m^3) | Number of moles | Temperature (deg C) |
|---|---|---|---|---|---|
| Start of intake stroke | 101,325.00 | 1.00000 | 0.0009000 | 0.039938541 | 15.00 |
| End of intake stroke | 96,258.75 | 0.95000 | 0.0010000 | 0.039938541 | 16.74 |

| The compression stroke, assumed to be adiabatic | | |
|---|---|---|
| P_start | 96,258.75 | N/m^2 |
| N_start | 0.0399385 | moles |
| T_start | 16.74 | degK |
| V_start | 0.0010000 | m^3 |
| Gamma | 1.340 | |
| P * (V^Gamma) | 9.1926393 | |
| DeltaV for graphing | 0.00002 | m^3 |

| Adiabatic Law | | Ideal Gas Law | | |
|---|---|---|---|---|
| Known | Unknown | Unknown | For graphing purposes only | |
| V | P | T (deg K) | P (atm) | T (degC) |
| 0.0010000 | 96,258.75 | 289.89318 | 0.95000 | 16.74318 |
| 0.0009800 | 98,900.22 | 291.89129 | 0.97607 | 18.74129 |
| 0.0009600 | 101,670.92 | 293.94480 | 1.00341 | 20.79480 |
| 0.0009400 | 104,580.06 | 296.05645 | 1.03212 | 22.90645 |
| 0.0009200 | 107,637.73 | 298.22918 | 1.06230 | 25.07918 |
| 0.0009000 | 110,854.99 | 300.46614 | 1.09405 | 27.31614 |
| 0.0008800 | 114,244.01 | 302.77073 | 1.12750 | 29.62073 |
| 0.0008600 | 117,818.18 | 305.14659 | 1.16277 | 31.99659 |
| 0.0008400 | 121,592.27 | 307.59767 | 1.20002 | 34.44767 |
| 0.0008200 | 125,582.65 | 310.12822 | 1.23940 | 36.97822 |
| 0.0008000 | 129,807.45 | 312.74286 | 1.28110 | 39.59286 |
| 0.0007800 | 134,286.84 | 315.44659 | 1.32531 | 42.29659 |
| 0.0007600 | 139,043.28 | 318.24484 | 1.37225 | 45.09484 |
| 0.0007400 | 144,101.90 | 321.14356 | 1.42218 | 47.99356 |
| 0.0007200 | 149,490.87 | 324.14920 | 1.47536 | 50.99920 |
| 0.0007000 | 155,241.86 | 327.26885 | 1.53212 | 54.11885 |
| 0.0006800 | 161,390.61 | 330.51028 | 1.59280 | 57.36028 |
| 0.0006600 | 167,977.59 | 333.88204 | 1.65781 | 60.73204 |
| 0.0006400 | 175,048.77 | 337.39357 | 1.72760 | 64.24357 |
| 0.0006200 | 182,656.59 | 341.05532 | 1.80268 | 67.90532 |
| 0.0006000 | 190,861.15 | 344.87886 | 1.88365 | 71.72886 |
| 0.0005800 | 199,731.57 | 348.87711 | 1.97120 | 75.72711 |
| 0.0005600 | 209,347.73 | 353.06451 | 2.06610 | 79.91451 |
| 0.0005400 | 219,802.48 | 357.45726 | 2.16928 | 84.30726 |
| 0.0005200 | 231,204.21 | 362.07360 | 2.28181 | 88.92360 |
| 0.0005000 | 243,680.30 | 366.93420 | 2.40494 | 93.78420 |
| 0.0004800 | 257,381.28 | 372.06257 | 2.54016 | 98.91257 |
| 0.0004600 | 272,486.33 | 377.48555 | 2.68923 | 104.33555 |
| 0.0004400 | 289,210.22 | 383.23405 | 2.85428 | 110.08405 |
| 0.0004200 | 307,812.44 | 389.34378 | 3.03787 | 116.19378 |
| 0.0004000 | 328,609.28 | 395.85634 | 3.24312 | 122.70634 |
| 0.0003800 | 351,989.90 | 402.82051 | 3.47387 | 129.67051 |
| 0.0003600 | 378,438.11 | 410.29398 | 3.73489 | 137.14398 |
| 0.0003400 | 408,562.47 | 418.34556 | 4.03220 | 145.19556 |
| 0.0003200 | 443,138.26 | 427.05814 | 4.37343 | 153.90814 |
| 0.0003000 | 483,167.52 | 436.53269 | 4.76849 | 163.38269 |
| 0.0002800 | 529,966.55 | 446.89374 | 5.23036 | 173.74374 |

| | | | | | |
|---|---|---|---|---|---|
| 0.0002600 | 585,296.51 | 458.29705 | 5.77643 | 185.14705 |
| 0.0002400 | 651,564.11 | 470.94065 | 6.43044 | 197.79065 |
| 0.0002200 | 732,139.48 | 485.08102 | 7.22565 | 211.93102 |
| 0.0002000 | 831,878.74 | 501.05777 | 8.21000 | 227.90777 |
| 0.0001800 | 958,021.07 | 519.33231 | 9.45493 | 246.18231 |
| 0.0001600 | 1,121,810.35 | 540.55166 | 11.07141 | 267.40166 |
| 0.0001400 | 1,341,617.30 | 565.65869 | 13.24073 | 292.50869 |
| 0.0001200 | 1,649,443.14 | 596.09623 | 16.27874 | 322.94623 |
| 0.0001000 | 2,105,911.99 | 634.21717 | 20.78374 | 361.06717 |

| | | |
|---|---|---|
| P_end | 2,105,911.99 | N/m^2 |
| N_end | 0.0399385 | moles |
| T_end | 634.22 | degK |
| V_end | 0.0001000 | m^3 |

**Heat released during combustion**

| | | |
|---|---|---|
| P_start | 2,105,911.99 | N/m^2 |
| N_start | 0.0399385 | moles |
| T_start | 634.22 | degK |
| V_start | 0.0001000 | m^3 |
| | | |
| Gasoline lower calorific value | 41.2 | MJ/kg of gasoline |
| Mass of octance in cylinder | 6.706332E-05 | kg |
| Heat released by combustion | 2,763.01 | Joules |

**Calculation of mixture composition after combustion**

| | Before combustion | After combustion | | | |
|---|---|---|---|---|---|
| Component gas | Number of molecules in cylinder | Number of molecules in cylinder | Number of moles in cylinder | Molecular weight (AMU) | Total mass (kg) |
| Nitrogen | 1.830582E+22 | 1.830582E+22 | 3.039752E-02 | 28.0134 | 8.515380E-04 |
| Oxygen | 4.419813E+21 | 0 | 0 | 31.9988 | 0.000000E+00 |
| Octane | 3.535850E+20 | 0 | 0 | 114.2200 | 0.000000E+00 |
| Argon | 2.189646E+20 | 2.189646E+20 | 3.635993E-04 | 39.9480 | 1.452506E-05 |
| Carbon dioxide | 3.216591E+20 | 3.150339E+21 | 5.231261E-03 | 44.0100 | 2.302278E-04 |
| Neon | 4.262074E+17 | 4.262074E+17 | 7.077339E-07 | 20.1790 | 1.428136E-08 |
| Helium | 1.228453E+17 | 1.228453E+17 | 2.039893E-07 | 4.0026 | 8.164877E-10 |
| Methane | 4.688750E+16 | 4.688750E+16 | 7.785852E-08 | 16.0440 | 1.249162E-09 |
| Water vapour | 4.311184E+20 | 3.613384E+21 | 6.000164E-03 | 18.0200 | 1.081230E-04 |
| Total for fuel-air mixture | 2.405155E+22 | 2.528910E+22 | 4.199354E-02 | | 1.204430E-03 |

| Temperature rise due to heat added, assumed to be Isochoric | | |
|---|---|---|
| Heat added by combustion | 2763.0089 | Joules |
| T_start | 634.22 | degK |
| V_start | 0.0001000 | m^3 |
| N_start | 4.1993537E-02 | moles |
| mass * CV_start (J/degK) | 1.058291 | Assumed to occur at a temerature of 650 degK |
| mass * CV_end (J/degK) | 1.434218 | Assumed to occur at a temerature of 2800 degK |
| mass * CV_average (J/degK) | 1.246254 | Effect is assumed to be linear |
| Trise | 2,217.05 | degK (Calculated by Trise = heat released / (mass x CV)) |
| T_end = T_start + Trise | 2,851.27 | degK |
| DeltaT for graphing | 44.34 | degK (=Trise / 50) |

| Ideal Gas Law | | | | |
|---|---|---|---|---|
| Knowns | | Unknown | For graphing purposes only | |
| T | V | P (N/m^2) | P (atm) | T (degC) |
| 634.22 | 0.0001000 | 2,214,269.49 | 21.85314 | 361.06717 |
| 678.56 | 0.0001000 | 2,369,079.17 | 23.38099 | 405.40819 |
| 722.90 | 0.0001000 | 2,523,888.85 | 24.90885 | 449.74920 |
| 767.24 | 0.0001000 | 2,678,698.53 | 26.43670 | 494.09022 |
| 811.58 | 0.0001000 | 2,833,508.21 | 27.96455 | 538.43123 |
| 855.92 | 0.0001000 | 2,988,317.89 | 29.49240 | 582.77225 |
| 900.26 | 0.0001000 | 3,143,127.57 | 31.02026 | 627.11327 |
| 944.60 | 0.0001000 | 3,297,937.25 | 32.54811 | 671.45428 |
| 988.95 | 0.0001000 | 3,452,746.93 | 34.07596 | 715.79530 |
| 1,033.29 | 0.0001000 | 3,607,556.61 | 35.60382 | 760.13631 |
| 1,077.63 | 0.0001000 | 3,762,366.29 | 37.13167 | 804.47733 |
| 1,121.97 | 0.0001000 | 3,917,175.98 | 38.65952 | 848.81834 |
| 1,166.31 | 0.0001000 | 4,071,985.66 | 40.18737 | 893.15936 |
| 1,210.65 | 0.0001000 | 4,226,795.34 | 41.71523 | 937.50037 |
| 1,254.99 | 0.0001000 | 4,381,605.02 | 43.24308 | 981.84139 |
| 1,299.33 | 0.0001000 | 4,536,414.70 | 44.77093 | 1026.18240 |
| 1,343.67 | 0.0001000 | 4,691,224.38 | 46.29878 | 1070.52342 |
| 1,388.01 | 0.0001000 | 4,846,034.06 | 47.82664 | 1114.86444 |
| 1,432.36 | 0.0001000 | 5,000,843.74 | 49.35449 | 1159.20545 |
| 1,476.70 | 0.0001000 | 5,155,653.42 | 50.88234 | 1203.54647 |
| 1,521.04 | 0.0001000 | 5,310,463.10 | 52.41020 | 1247.88748 |
| 1,565.38 | 0.0001000 | 5,465,272.78 | 53.93805 | 1292.22850 |
| 1,609.72 | 0.0001000 | 5,620,082.46 | 55.46590 | 1336.56951 |
| 1,654.06 | 0.0001000 | 5,774,892.14 | 56.99375 | 1380.91053 |
| 1,698.40 | 0.0001000 | 5,929,701.82 | 58.52161 | 1425.25154 |
| 1,742.74 | 0.0001000 | 6,084,511.50 | 60.04946 | 1469.59256 |
| 1,787.08 | 0.0001000 | 6,239,321.18 | 61.57731 | 1513.93357 |
| 1,831.42 | 0.0001000 | 6,394,130.86 | 63.10517 | 1558.27459 |
| 1,875.77 | 0.0001000 | 6,548,940.54 | 64.63302 | 1602.61560 |
| 1,920.11 | 0.0001000 | 6,703,750.22 | 66.16087 | 1646.95662 |
| 1,964.45 | 0.0001000 | 6,858,559.90 | 67.68872 | 1691.29764 |
| 2,008.79 | 0.0001000 | 7,013,369.58 | 69.21658 | 1735.63865 |
| 2,053.13 | 0.0001000 | 7,168,179.26 | 70.74443 | 1779.97967 |
| 2,097.47 | 0.0001000 | 7,322,988.94 | 72.27228 | 1824.32068 |

| | | | | | |
|---|---|---|---|---|---|
| 2,141.81 | 0.0001000 | 7,477,798.62 | 73.80013 | 1868.66170 |
| 2,186.15 | 0.0001000 | 7,632,608.30 | 75.32799 | 1913.00271 |
| 2,230.49 | 0.0001000 | 7,787,417.98 | 76.85584 | 1957.34373 |
| 2,274.83 | 0.0001000 | 7,942,227.66 | 78.38369 | 2001.68474 |
| 2,319.18 | 0.0001000 | 8,097,037.34 | 79.91155 | 2046.02576 |
| 2,363.52 | 0.0001000 | 8,251,847.02 | 81.43940 | 2090.36677 |
| 2,407.86 | 0.0001000 | 8,406,656.70 | 82.96725 | 2134.70779 |
| 2,452.20 | 0.0001000 | 8,561,466.38 | 84.49510 | 2179.04881 |
| 2,496.54 | 0.0001000 | 8,716,276.06 | 86.02296 | 2223.38982 |
| 2,540.88 | 0.0001000 | 8,871,085.74 | 87.55081 | 2267.73084 |
| 2,585.22 | 0.0001000 | 9,025,895.42 | 89.07866 | 2312.07185 |
| 2,629.56 | 0.0001000 | 9,180,705.10 | 90.60651 | 2356.41287 |
| 2,673.90 | 0.0001000 | 9,335,514.78 | 92.13437 | 2400.75388 |
| 2,718.24 | 0.0001000 | 9,490,324.46 | 93.66222 | 2445.09490 |
| 2,762.59 | 0.0001000 | 9,645,134.14 | 95.19007 | 2489.43591 |
| 2,806.93 | 0.0001000 | 9,799,943.82 | 96.71793 | 2533.77693 |
| 2,851.27 | 0.0001000 | 9,954,753.50 | 98.24578 | 2578.11794 |

| | | |
|---|---|---|
| P_end | 9,954,753.50 | N/m^2 |
| N_end | 0.0419935 | moles |
| T_end | 2,851.27 | degK |
| V_end | 0.0001000 | m^3 |

| Power stroke, assumed to be adiabatic | | |
|---|---|---|
| P_start | 9,954,753.50 | N/m^2 |
| N_start | 0.0419935 | moles |
| T_start | 2,851.27 | degK |
| V_start | 0.0001000 | m^3 |
| Gamma | 1.225 | |
| P * (V^Gamma) | 125.3229 | |
| DeltaV for graphing | 0.0000200 | m^3 |

| Adiabatic Law | | Ideal Gas Law | | |
|---|---|---|---|---|
| Known | Unknown | Unknown | For graphing purposes only | |
| V | P | T (deg K) | P (atm) | T (degC) |
| 0.0001000 | 9,954,753.50 | 2,851.26794 | 98.24578 | 2578.11794 |
| 0.0001200 | 7,962,207.37 | 2,736.66886 | 78.58088 | 2463.51886 |
| 0.0001400 | 6,592,098.18 | 2,643.37771 | 65.05895 | 2370.22771 |
| 0.0001600 | 5,597,363.76 | 2,565.13978 | 55.24169 | 2291.98978 |
| 0.0001800 | 4,845,311.38 | 2,498.05340 | 47.81951 | 2224.90340 |
| 0.0002000 | 4,258,618.79 | 2,439.53067 | 42.02930 | 2166.38067 |
| 0.0002200 | 3,789,332.60 | 2,387.77240 | 37.39781 | 2114.62240 |
| 0.0002400 | 3,406,212.51 | 2,341.48027 | 33.61670 | 2068.33027 |
| 0.0002600 | 3,088,077.27 | 2,299.68858 | 30.47695 | 2026.53858 |
| 0.0002800 | 2,820,083.21 | 2,261.66082 | 27.83206 | 1988.51082 |
| 0.0003000 | 2,591,534.37 | 2,226.82326 | 25.57646 | 1953.67326 |
| 0.0003200 | 2,394,538.30 | 2,194.72083 | 23.63226 | 1921.57083 |
| 0.0003400 | 2,223,150.36 | 2,164.98691 | 21.94079 | 1891.83691 |
| 0.0003600 | 2,072,812.16 | 2,137.32206 | 20.45707 | 1864.17206 |
| 0.0003800 | 1,939,972.64 | 2,111.47879 | 19.14604 | 1838.32879 |
| 0.0004000 | 1,821,826.53 | 2,087.25031 | 17.98003 | 1814.10031 |
| 0.0004200 | 1,716,129.79 | 2,064.46223 | 16.93688 | 1791.31223 |
| 0.0004400 | 1,621,067.06 | 2,042.96619 | 15.99869 | 1769.81619 |
| 0.0004600 | 1,535,154.77 | 2,022.63501 | 15.15080 | 1749.48501 |
| 0.0004800 | 1,457,169.24 | 2,003.35887 | 14.38114 | 1730.20887 |
| 0.0005000 | 1,386,092.63 | 1,985.04237 | 13.67967 | 1711.89237 |
| 0.0005200 | 1,321,071.78 | 1,967.60211 | 13.03796 | 1694.45211 |
| 0.0005400 | 1,261,386.43 | 1,950.96481 | 12.44892 | 1677.81481 |
| 0.0005600 | 1,206,424.58 | 1,935.06575 | 11.90648 | 1661.91575 |
| 0.0005800 | 1,155,663.03 | 1,919.84750 | 11.40551 | 1646.69750 |
| 0.0006000 | 1,108,651.96 | 1,905.25891 | 10.94154 | 1632.10891 |
| 0.0006200 | 1,065,002.66 | 1,891.25418 | 10.51076 | 1618.10418 |
| 0.0006400 | 1,024,377.53 | 1,877.79223 | 10.10982 | 1604.64223 |
| 0.0006600 | 986,482.06 | 1,864.83601 | 9.73582 | 1591.68601 |
| 0.0006800 | 951,058.20 | 1,852.35203 | 9.38621 | 1579.20203 |
| 0.0007000 | 917,878.95 | 1,840.30994 | 9.05876 | 1567.15994 |
| 0.0007200 | 886,743.89 | 1,828.68213 | 8.75148 | 1555.53213 |
| 0.0007400 | 857,475.37 | 1,817.44340 | 8.46262 | 1544.29340 |
| 0.0007600 | 829,915.47 | 1,806.57075 | 8.19063 | 1533.42075 |
| 0.0007800 | 803,923.31 | 1,796.04307 | 7.93411 | 1522.89307 |
| 0.0008000 | 779,372.86 | 1,785.84098 | 7.69181 | 1512.69098 |
| 0.0008200 | 756,151.02 | 1,775.94665 | 7.46263 | 1502.79665 |

| | | | | | |
|---|---|---|---|---|---|
| | 0.0008400 | 734,156.06 | 1,766.34361 | 7.24556 | 1493.19361 |
| | 0.0008600 | 713,296.20 | 1,757.01666 | 7.03969 | 1483.86666 |
| | 0.0008800 | 693,488.46 | 1,747.95171 | 6.84420 | 1474.80171 |
| | 0.0009000 | 674,657.63 | 1,739.13569 | 6.65835 | 1465.98569 |
| | 0.0009200 | 656,735.39 | 1,730.55645 | 6.48147 | 1457.40645 |
| | 0.0009400 | 639,659.55 | 1,722.20269 | 6.31295 | 1449.05269 |
| | 0.0009600 | 623,373.38 | 1,714.06388 | 6.15222 | 1440.91388 |
| | 0.0009800 | 607,825.01 | 1,706.13018 | 5.99877 | 1432.98018 |
| | 0.0010000 | 592,966.98 | 1,698.39238 | 5.85213 | 1425.24238 |

| | | |
|---|---|---|
| P_end | 592,966.98 | N/m^2 |
| N_end | 0.0419935 | moles |
| T_end | 1,698.39 | degK |
| V_end | 0.0010000 | m^3 |

**Exhaust stroke for the 10% of the gas which is retained, assumed to be Isochoric**

Total gas inside cylinder

| | | |
|---|---|---|
| P_start | 592,966.98 | N/m^2 |
| N_start | 0.0419935 | moles |
| T_start | 1,698.39 | degK |
| V_start | 0.0010000 | m^3 |
| | | |
| Number of moles exhausted | 0.0377942 | = 90% |
| Number of moles retained | 0.0041994 | = 10% |

Starting conditions for the retained gas

| | | |
|---|---|---|
| P_start | 592,966.98 | N/m^2 |
| N_start | 0.0041994 | moles |
| T_start | 1,698.39 | degK |
| V_start (10% of cylinder) | 0.0001000 | m^3 |

Ending conditions for the retained gas

| | | |
|---|---|---|
| P_end (N/m^2) | 106,391.25 | = Ambient pressure plus exhaust overpressure |
| N_end | 0.0041994 | moles |
| V_end (m^3) | 0.0001000 | = Minimum cylinder volume |
| T_end (degK) | 304.72875 | Using the Ideal Gas Law with number of moles retained |

Heat lost by the retained gas

| | | |
|---|---|---|
| T_fall = T_start - T_end | 1,393.66 | degK |
| Total mass * CV_start (J/degK) | 1.337443 | Table entry at 1700 degK for total mass in cylinder |
| Total mass * CV_end (J/degK) | 0.940395 | Table entry at 300degK for total mass in cylinder |
| Total mass * CV_avg (J/degK) | 1.138919 | Average of table entries for total mass |
| Retained mass * CV_avg | 0.113892 | Average of table entries for retained mass |
| Heat lost by retained gas only | 158.73 | Joules (Heat lost = mass x CV x Tfall) |
| | | |
| DeltaT for graphing | 27.87 | degK (=Tfall / 50) |

| Ideal Gas Law | | | | |
|---|---|---|---|---|
| Knowns | | Unknown | For graphing purposes only | |
| T | V | P (N/m^2) | P (atm) | T (degC) |
| 1,698.39 | 0.0010000 | 592,966.98 | 5.85213 | 1425.24238 |
| 1,670.52 | 0.0010000 | 583,235.47 | 5.75609 | 1397.36910 |
| 1,642.65 | 0.0010000 | 573,503.95 | 5.66004 | 1369.49583 |
| 1,614.77 | 0.0010000 | 563,772.44 | 5.56400 | 1341.62256 |
| 1,586.90 | 0.0010000 | 554,040.92 | 5.46796 | 1313.74929 |
| 1,559.03 | 0.0010000 | 544,309.41 | 5.37192 | 1285.87601 |
| 1,531.15 | 0.0010000 | 534,577.89 | 5.27587 | 1258.00274 |
| 1,503.28 | 0.0010000 | 524,846.38 | 5.17983 | 1230.12947 |
| 1,475.41 | 0.0010000 | 515,114.86 | 5.08379 | 1202.25620 |
| 1,447.53 | 0.0010000 | 505,383.35 | 4.98775 | 1174.38292 |
| 1,419.66 | 0.0010000 | 495,651.83 | 4.89170 | 1146.50965 |
| 1,391.79 | 0.0010000 | 485,920.32 | 4.79566 | 1118.63638 |
| 1,363.91 | 0.0010000 | 476,188.81 | 4.69962 | 1090.76311 |
| 1,336.04 | 0.0010000 | 466,457.29 | 4.60358 | 1062.88983 |

| | | | | |
|---|---|---|---|---|
| 1,308.17 | 0.0010000 | 456,725.78 | 4.50753 | 1035.01656 |
| 1,280.29 | 0.0010000 | 446,994.26 | 4.41149 | 1007.14329 |
| 1,252.42 | 0.0010000 | 437,262.75 | 4.31545 | 979.27002 |
| 1,224.55 | 0.0010000 | 427,531.23 | 4.21941 | 951.39674 |
| 1,196.67 | 0.0010000 | 417,799.72 | 4.12336 | 923.52347 |
| 1,168.80 | 0.0010000 | 408,068.20 | 4.02732 | 895.65020 |
| 1,140.93 | 0.0010000 | 398,336.69 | 3.93128 | 867.77693 |
| 1,113.05 | 0.0010000 | 388,605.17 | 3.83523 | 839.90365 |
| 1,085.18 | 0.0010000 | 378,873.66 | 3.73919 | 812.03038 |
| 1,057.31 | 0.0010000 | 369,142.14 | 3.64315 | 784.15711 |
| 1,029.43 | 0.0010000 | 359,410.63 | 3.54711 | 756.28384 |
| 1,001.56 | 0.0010000 | 349,679.12 | 3.45106 | 728.41056 |
| 973.69 | 0.0010000 | 339,947.60 | 3.35502 | 700.53729 |
| 945.81 | 0.0010000 | 330,216.09 | 3.25898 | 672.66402 |
| 917.94 | 0.0010000 | 320,484.57 | 3.16294 | 644.79075 |
| 890.07 | 0.0010000 | 310,753.06 | 3.06689 | 616.91747 |
| 862.19 | 0.0010000 | 301,021.54 | 2.97085 | 589.04420 |
| 834.32 | 0.0010000 | 291,290.03 | 2.87481 | 561.17093 |
| 806.45 | 0.0010000 | 281,558.51 | 2.77877 | 533.29766 |
| 778.57 | 0.0010000 | 271,827.00 | 2.68272 | 505.42438 |
| 750.70 | 0.0010000 | 262,095.48 | 2.58668 | 477.55111 |
| 722.83 | 0.0010000 | 252,363.97 | 2.49064 | 449.67784 |
| 694.95 | 0.0010000 | 242,632.45 | 2.39460 | 421.80457 |
| 667.08 | 0.0010000 | 232,900.94 | 2.29855 | 393.93129 |
| 639.21 | 0.0010000 | 223,169.43 | 2.20251 | 366.05802 |
| 611.33 | 0.0010000 | 213,437.91 | 2.10647 | 338.18475 |
| 583.46 | 0.0010000 | 203,706.40 | 2.01043 | 310.31148 |
| 555.59 | 0.0010000 | 193,974.88 | 1.91438 | 282.43820 |
| 527.71 | 0.0010000 | 184,243.37 | 1.81834 | 254.56493 |
| 499.84 | 0.0010000 | 174,511.85 | 1.72230 | 226.69166 |
| 471.97 | 0.0010000 | 164,780.34 | 1.62626 | 198.81839 |
| 444.10 | 0.0010000 | 155,048.82 | 1.53021 | 170.94511 |
| 416.22 | 0.0010000 | 145,317.31 | 1.43417 | 143.07184 |
| 388.35 | 0.0010000 | 135,585.79 | 1.33813 | 115.19857 |
| 360.48 | 0.0010000 | 125,854.28 | 1.24209 | 87.32530 |
| 332.60 | 0.0010000 | 116,122.76 | 1.14604 | 59.45202 |
| 304.73 | 0.0010000 | 106,391.25 | 1.05000 | 31.57875 |

| | | |
|---|---|---|
| P_end | 106,391.25 | N/m^2 |
| N_end | 0.0419935 | moles |
| T_end | 304.73 | degK |
| V_end | 0.0010000 | m^3 |

| Exhaust stroke for the 90% of the gas which leaves the cylinder | | |
|---|---|---|
| Starting conditions for the exhausted gas | | |
| P_start | 592,966.98 | N/m^2 |
| N_start | 0.0377942 | moles |
| T_start | 1,698.39 | degK |
| V_start (90% of cylinder) | 0.0009000 | m^3 |
| | | |
| Volume after mixing with the ambient air | | |
| P_end | 101,325.00 | N/m^2 |
| N_end | 0.0377942 | moles |
| T_end | 288.15 | degK |
| V_end | 0.0008936 | Using the Ideal Gas Law |
| | | |
| Step #1 - Adiabatic adjustment of pressure as volume changes from V_start to V_end | | |
| Gamma | 1.200 | |
| P_start * (V_start^Gamma) | 131.2567 | |
| P_adj * (V_end*Gamma) | 131.2567 | No change since adiabatic process |
| P_adj | 598,077.03 | |
| | | |
| Step #2 - Calculate the temperature after the adiabatic adjustment | | |
| T_adj (degK) | 1,700.82 | Using the Ideal Gas Law |
| | | |
| Step #2 - Heat lost by the exhausted gas | | |
| T_fall = T_adj - T_end | 1,412.67 | degK |
| Total mass * CV_start (J/degK) | 1.337443 | Table entry at 1700 degK for total mass in cylinder |
| Total mass * CV_end (J/degK) | 0.940395 | Table entry at 300degK for total mass in cylinder |
| Total mass * CV_avg (J/degK) | 1.138919 | Average of table entries for total mass |
| Exhausted mass * CV_avg | 1.025027 | Average of table entries for exhausted mass |
| Heat lost by exhausted gas | 1,448.03 | Joules (Heat lost = mass x CV x Tfall) |

| Thermal work done | | |
|---|---|---|
| Heat released by combustion | 2,763.01 | Joules |
| Heat lost in exhausted gas | 1,448.03 | Joules |
| Heat lost by retained gas | 158.73 | Joules |
| Net work done (A) | 1,156.25 | Joules |
| Ideal thermal efficiency | 41.848% | = Net work done / heat released by combustion |
| | | |
| Sample engine | | |
| Nominal volume | 2.5 | liters |
| Number of cylinders | 4 | |
| Engine rotational speed | 2,000 | RPM |
| | | |
| Volume of each cylinder (B) | 0.625 | liters |
| Engine rotational speed | 33.33333333 | revolutions/sec |
| Power strokes/sec/cylinder | 16.66666667 | power strokes/sec/cylinder |
| Power strokes/sec (C) | 66.66666667 | total power strokes/sec |
| Net work done/sec (A x B x C) | 48,177.24 | Joules/sec |
| Power (Watts) | 48,177.24 | Watts |
| Power (HP) | 64.61 | HP |

| Real air consumed per minute | | | | |
|---|---|---|---|---|
| | Per liter of cylinder volume | | | |
| Component gas | Number of new molecules drawn in | Number of new moles drawn in | Molecular weight (AMU) | Total mass (kg) |
| Nitrogen | 1.647524E+22 | 2.735777E-02 | 28.0134 | 7.663842E-04 |
| Oxygen | 4.419813E+21 | 7.339271E-03 | 31.9988 | 2.348479E-04 |
| Argon | 1.970682E+20 | 3.272394E-04 | 39.9480 | 1.307256E-05 |
| Carbon dioxide | 6.625204E+18 | 1.100141E-05 | 44.0100 | 4.841720E-07 |
| Neon | 3.835866E+17 | 6.369606E-07 | 20.1790 | 1.285323E-08 |
| Helium | 1.105607E+17 | 1.835904E-07 | 4.0026 | 7.348389E-10 |
| Methane | 4.219875E+16 | 7.007267E-08 | 16.0440 | 1.124246E-09 |
| Water vapour | 6.978007E+19 | 1.158725E-04 | 18.0200 | 2.088023E-06 |
| Total for fuel-air mixture | 2.116906E+22 | 3.515205E-02 | | 1.016892E-03 |

| | | |
|---|---|---|
| Mass real air per liter (E) | 1.0168915E-03 | kg/power stroke |
| Real air/sec (E x B x C) | 4.2370480E-02 | kg/sec |
| Real air/min (kg) | 2.5422 | kg/min |
| Real air/min (lb) | 5.5929 | lb/min |

| Dry air consumed per minute | | | | |
|---|---|---|---|---|
| | Per liter of cylinder volume | | | |
| Component gas | Number of new molecules drawn in | Number of new moles drawn in | Molecular weight (AMU) | Total mass (kg) |
| Nitrogen | 1.647524E+22 | 2.735777E-02 | 28.0134 | 7.663842E-04 |
| Oxygen | 4.419813E+21 | 7.339271E-03 | 31.9988 | 2.348479E-04 |
| Argon | 1.970682E+20 | 3.272394E-04 | 39.9480 | 1.307256E-05 |
| Carbon dioxide | 6.625204E+18 | 1.100141E-05 | 44.0100 | 4.841720E-07 |
| Neon | 3.835866E+17 | 6.369606E-07 | 20.1790 | 1.285323E-08 |
| Helium | 1.105607E+17 | 1.835904E-07 | 4.0026 | 7.348389E-10 |
| Methane | 4.219875E+16 | 7.007267E-08 | 16.0440 | 1.124246E-09 |
| Water vapour | | | 18.0200 | |
| Total for fuel-air mixture | 2.109928E+22 | 3.503617E-02 | | 1.014803E-03 |

| | | |
|---|---|---|
| Mass dry air per liter (F) | 1.0148035E-03 | kg/power stroke |
| Dry air/sec (F x B x C) | 4.2283479E-02 | kg/sec |
| Dry air/min (kg) | 2.5370 | kg/min |
| Dry air/min (lb) | 5.5814 | lb/min |

| Thermal work done versus air consumed, no radiation | | |
|---|---|---|
| HP / lb/min of real air | 11.5516 | HP/ lb/min |
| HP / lb/min of dry air | 11.5753 | HP/ lb/min |
| Average | 11.5634 | HP/ lb/min |

| Mechanial work done versus air consumed, no radiation | | |
|---|---|---|
| Thermal work (average) | 11.5634 | HP/ lb/min |
| Typical mechanical efficiency | 94% | |
| Mechanical work | 10.8696 | HP/ lb/min |

| Take into account heat lost from the engine's surface | | |
|---|---|---|
| Net work done/sec (A x B x C) | 48,177.24 | Joules/sec |
| Less: Heat lost by engine | 5,000.00 | |
| Net useful work done | 43,177.24 | |
| Useful power (Watts) | 43,177.24 | Watts |
| Useful power (HP) | 57.90 | HP |

Useful power versus air consumed, accounting for heat lost from engine's surface

| | | |
|---|---|---|
| Useful HP / lb/min of real air | 10.3527 | HP/ lb/min |
| Useful HP / lb/min of dry air | 10.3740 | HP/ lb/min |
| Average | 10.3633 | |

Useful mechanical power versus air consumed, accounting for heat lost from engine's surface

| | | |
|---|---|---|
| Useful power (average) | 10.3633 | HP/ lb/min |
| Typical mechanical efficiency | 94% | |
| Mechanical work | 9.7415 | HP/ lb/min |